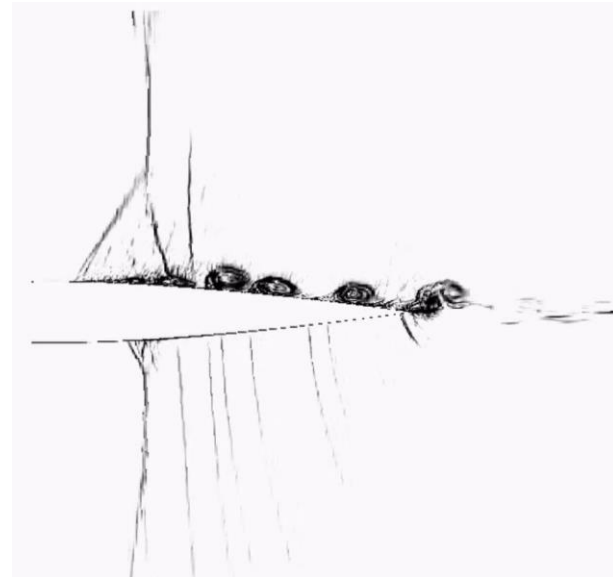
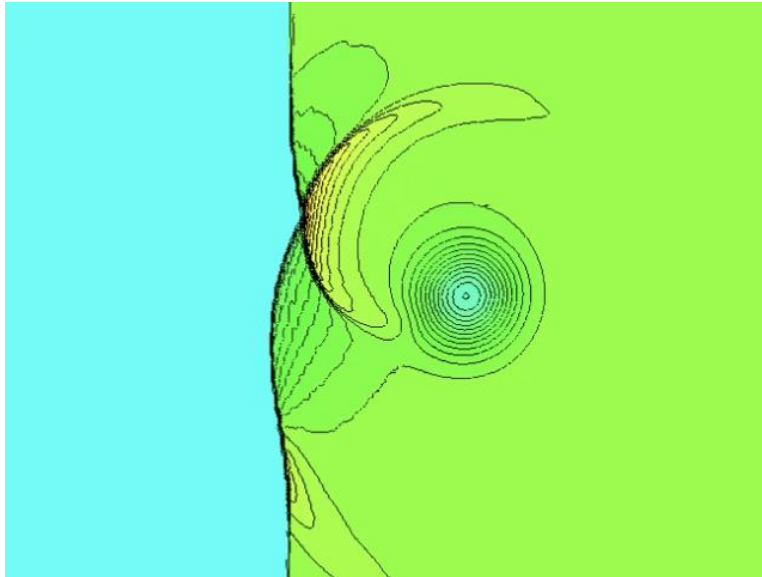


Shock capturing in SU2 DG-FEM solver



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Outline

- Motivation for high-order methods
- SU2 DG-FEM solver
- Shock capturing in DG
- Proposed shock detector and filtering
- Results
- Future work

Motivation for high-order methods

- Success of 2nd order numerical methods

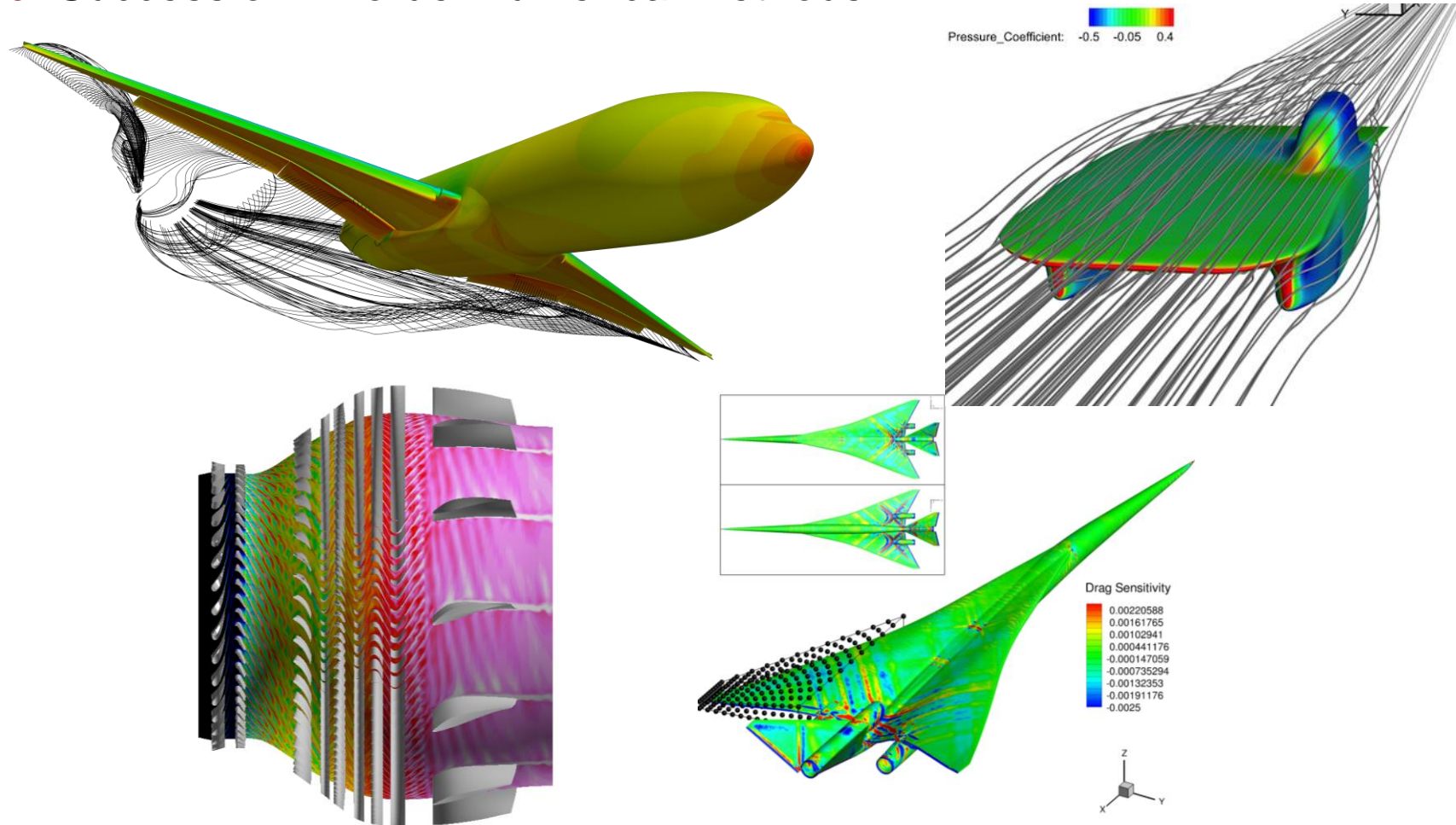
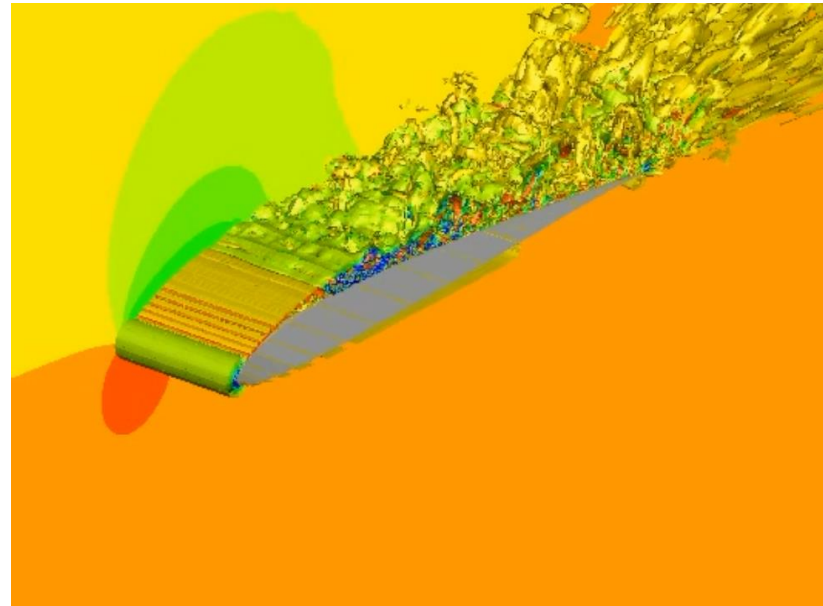


Image sources: Aerospace Design Lab, <https://su2code.github.io/>

Motivation for high-order methods

○ But...

- 2nd order accuracy may not be sufficient for some applications
- Examples: wake and vortex flows, noise prediction, LES/DNS

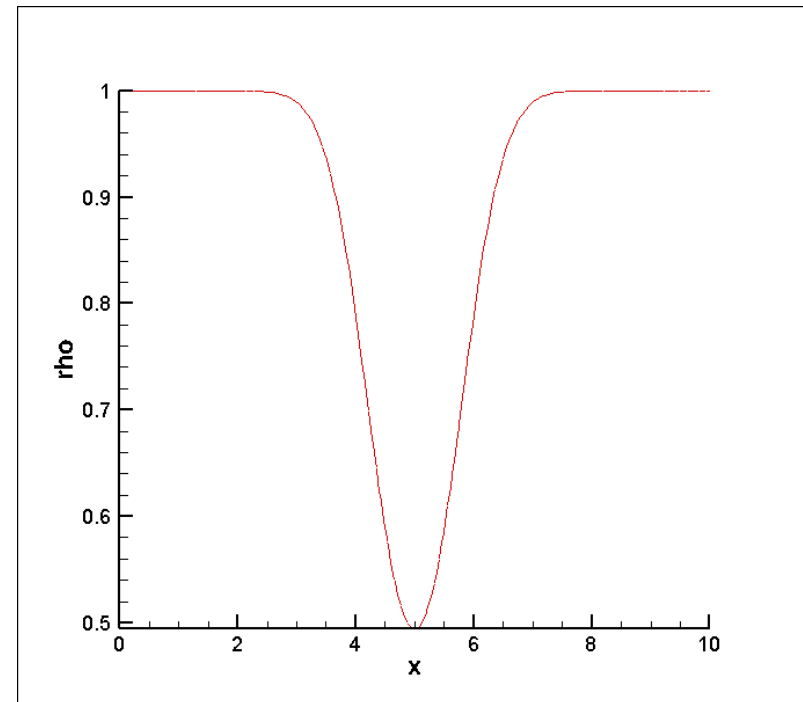
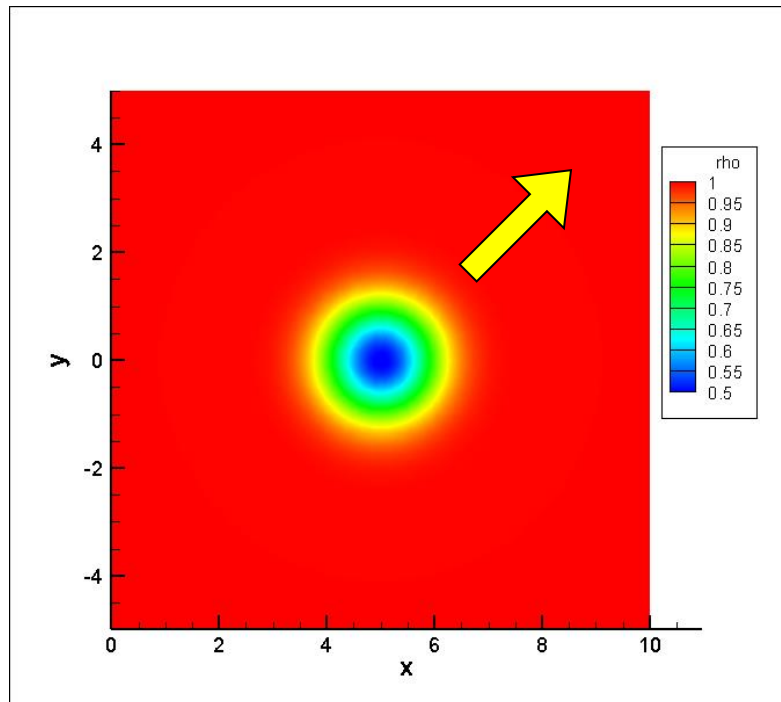


Implicit LES, SD 7003, $Re = 60,000$
 $p = 4$, Hexahedra, SU2 DG-FEM
solver

Image source (left): National Renewable Energy Lab

Motivation for high-order methods

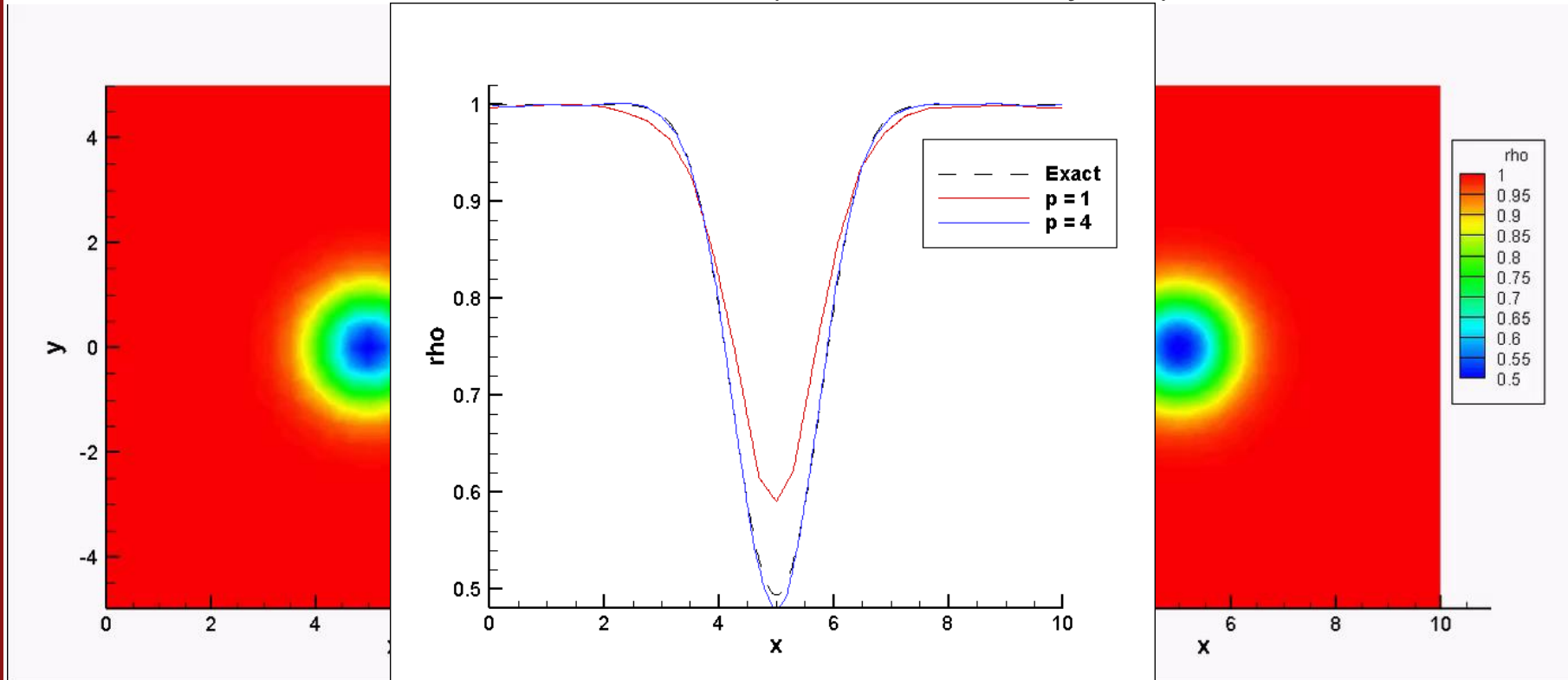
- Fundamental example: Isentropic vortex problem
 - Euler equations, discontinuous Galerkin method, ADER
 - Domain : $[0, 10] \times [-5, 5]$ / Periodic boundary conditions
 - $\rho = 1$, $p = 1$, $\Delta\rho = 0.5$
 - $(u, v) = (1, 1)$: Diagonal flow



ρ vs. x @ $y = 0.0$

Motivation for high-order methods

- Fundamental example: Isentropic vortex problem
 - Run simulations until $t = 100.0$ (10 convective cycles)



$p = 1$, nElem = 1,280, nDOFs = 3,840

Relative error, $L_\infty = 11.0\%$

$p = 4$, nElem = 158, nDOFs = 2,370

Relative error, $L_\infty = 2.26\%$

SU2 DG-FEM Solver

- Both 2D and 3D
- All standard elements (tri, quad, tet, pyra, prism, hex)
- Curved elements of arbitrary order
- Polynomial order can differ in individual elements
- Explicit time integration schemes (Runge-Kutta type)
- Time-accurate local time stepping via ADER-DG
- Task scheduling approach for efficient parallelization
- Preliminary implementation of LES models and shock capturing

Shock capturing in DG

- Goal: Simultaneously capture a discontinuity robustly and preserve accuracy (both error constant and/or asymptotic rate)
- Shock capturing comprises two components
 - Detecting a discontinuity
 - Resolving a discontinuity
- Detecting a discontinuity
 - Based on local values (Persson et al., Klockner et al.)
 - Based on local values and direct neighbors (Lv et al.)
 - Based on local values and Voronoi neighbors (Park et al., Clain et al.)
- Resolving a discontinuity
 - A priori method (Artificial viscosity, Limiting)
 - A posteriori method (Sub-cell finite volume limiter)

In order to preserve HPC-favorable characteristics (locality) it is critical to develop a detection method that only relies on local values

Proposed shock detector and filtering

○ Shock detector^[1]

- $\phi^n = |\hat{u}_N^n|_2 / |\hat{u}_0^n|_2$
 - $\beta = \phi^{n+1} / \phi^n$
- where \hat{u}_N^n : modal coefficient of the highest polynomial at t_n
 \hat{u}_0^n : modal coefficient of the lowest polynomial at t_n

○ Shock detection steps

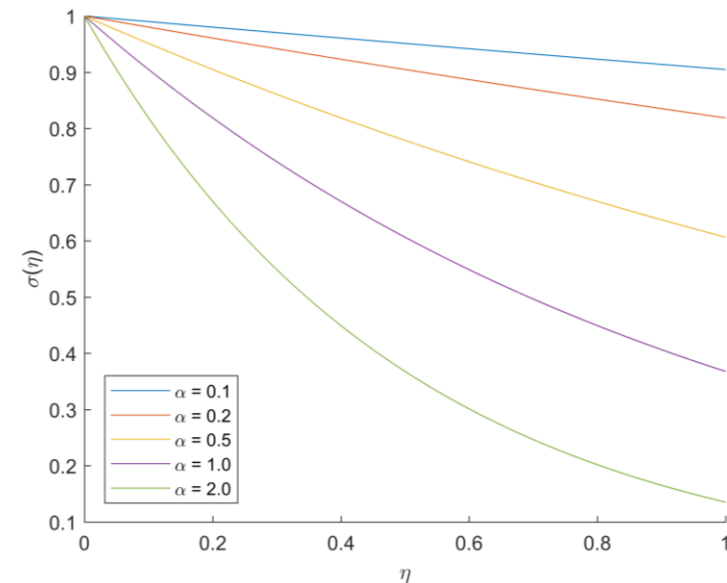
- 1) Calculate ϕ^n from $u(\vec{x}, t_n)$
- 2) March forward one time-step and calculate candidate solution $u(\vec{x}, t_{n+1})$
- 3) Calculate ϕ^{n+1} from $u(\vec{x}, t_{n+1})$
- 4) If $\phi^{n+1} < \phi_0$, an element is not a *troubled* element. Else, the current element might be a *troubled* element
- 5) For elements with $\phi^{n+1} \geq \phi_0$, calculate β . If $\beta \geq \beta_0$, then the current element is a *troubled* element

[1] Jae Hwan Choi, Juan J. Alonso, and Edwin van der Weide. "Simple shock detector for discontinuous Galerkin method", AIAA Scitech 2019 Forum

Proposed shock detector and filtering

- Filtering method^[1]
 - Effectively same as using artificial viscosity
 - Does not restrict time-step size
 - Implementation becomes a simple matrix-vector multiplication
- Second-order exponential filter
 - $\sigma(\eta) = \exp(-\alpha\eta^2)$ where $\eta = \min(n/N, 1.0)$
 - n is the polynomial order of a basis function
 - α is a parameter to change the strength of the filter
 - σ values are multiplication factors for the modal coefficients, \hat{u} , of the element to obtain filtered modal coefficients, \tilde{u} .

$$\tilde{u} = \sigma \hat{u}$$



[1] Hesthaven, J. S., and Warburton, T., "Nodal Discontinuous Galerkin Methods: Algorithms, Analysis, and Applications," Springer-Verlag New York, 2008, Chaps. 3, 5.

Proposed shock detector and filtering

○ Relationship between ϕ^{n+1} and α [1]

• 2D, triangular element

N = 2

$$\begin{aligned}\alpha(\phi^{n+1}) &= (6.18\phi^{n+1} + 0.04383) & 0.00200 \leq \phi^{n+1} < 0.01980 \\ &= (2.93\phi^{n+1} + 0.10818) & 0.01980 \leq \phi^{n+1}\end{aligned}$$

N = 3

$$\alpha(\phi^{n+1}) = (35.7\phi^{n+1} + 0.11362) \quad 0.00050 \leq \phi^{n+1}$$

N = 4

$$\alpha(\phi^{n+1}) = (38.0\phi^{n+1} + 0.07520) \quad 0.00010 \leq \phi^{n+1}$$

• 2D, quadrilateral element

N = 2

$$\begin{aligned}\alpha(\phi^{n+1}) &= (3.67\phi^{n+1} + 0.09004) & 0.01000 \leq \phi^{n+1} < 0.01916 \\ &= (2.46\phi^{n+1} + 0.11323) & 0.01916 \leq \phi^{n+1}\end{aligned}$$

N = 3

$$\alpha(\phi^{n+1}) = (20.54\phi^{n+1} + 0.025434) \quad 0.00500 \leq \phi^{n+1}$$

N = 4

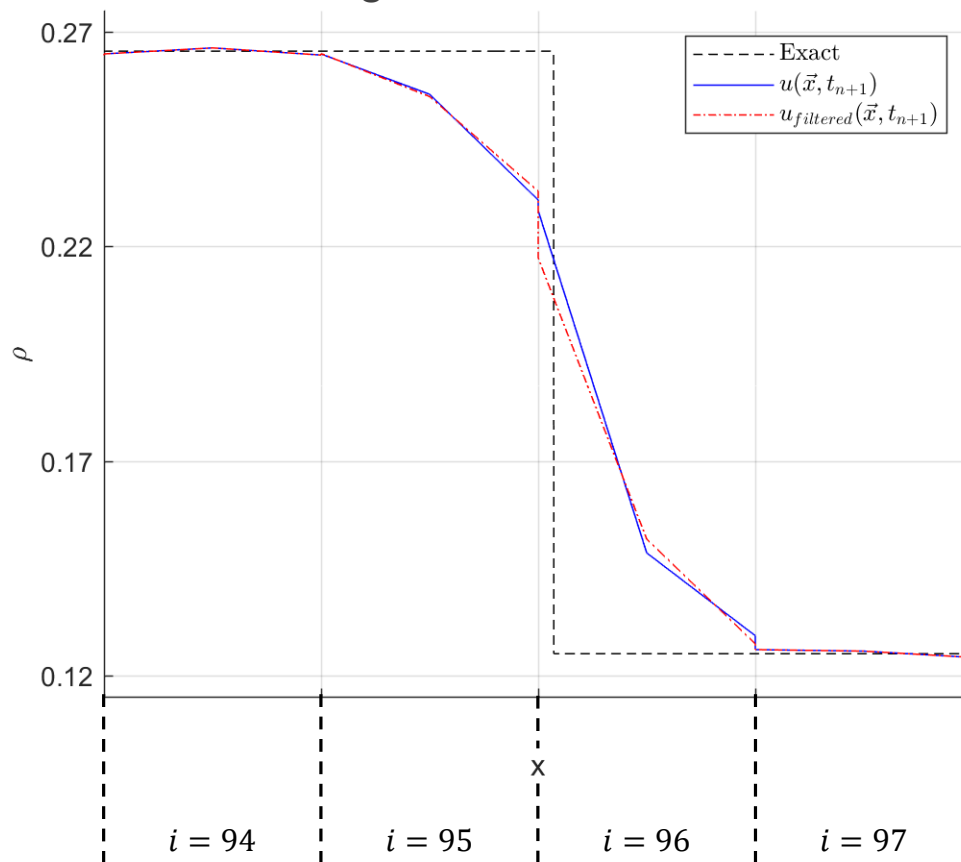
$$\alpha(\phi^{n+1}) = (30.15\phi^{n+1} + 0.093116) \quad 0.00200 \leq \phi^{n+1}$$

[1] Jae Hwan Choi, Juan J. Alonso, and Edwin van der Weide. "Simple shock detector for discontinuous Galerkin method", AIAA Scitech 2019 Forum

Proposed shock detector and filtering

Detection steps

- 1) Calculate $\rho(\vec{x}, t_{n+1})$ and $\rho(\vec{x}, t_n)$ at each element i . Filter the current element i if $\rho(\vec{x}, t_{n+1}) < \rho(\vec{x}, t_n)$.

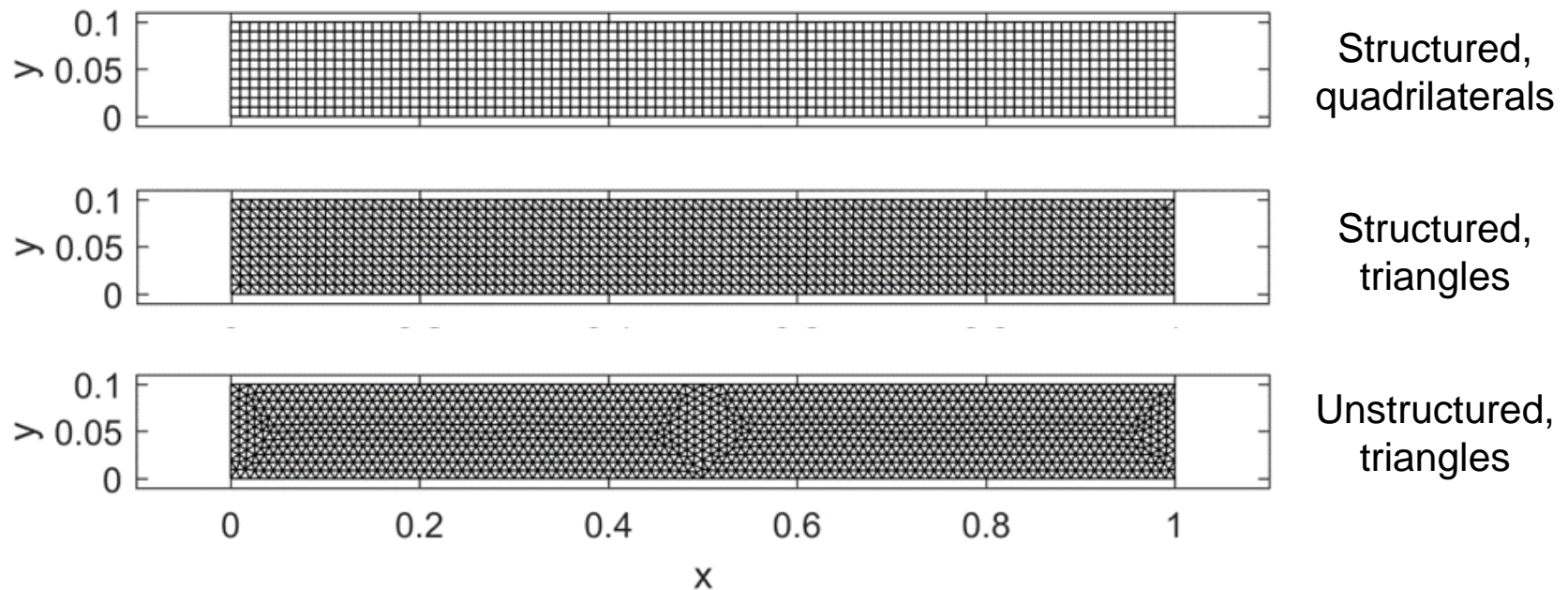


$\phi_0 \equiv 0.00400$ for $i=97$, $\rho=2$
 element $\beta_0 = 1.00$

Results

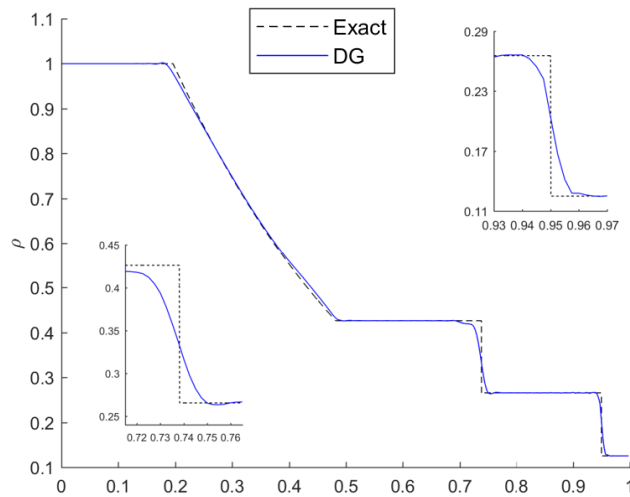
○ 2D shock tube problem

- $p_L/p_R = 10$
- $CFL = 0.45$
- Each simulation runs until the right running shock wave arrives $x = 0.95$
- $\Omega = [0, 1] \times [0, 0.1]$. The domain is discretized with triangles or quadrilaterals of characteristic lengths 0.01

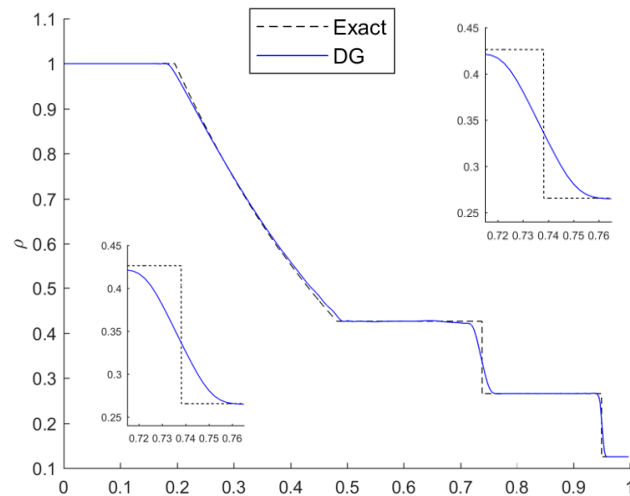


Results

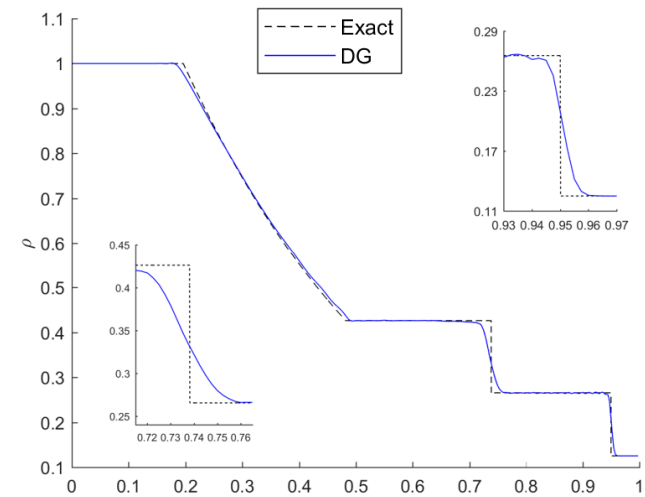
○ 2D shock tube problem



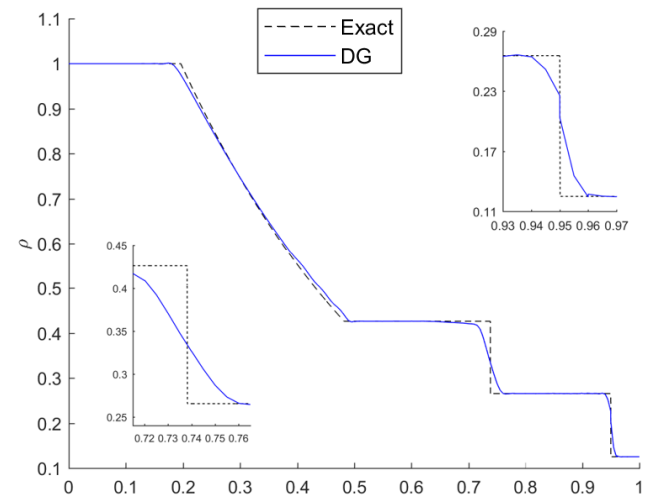
$p = 2, \Delta l = 0.01$ / Structured, quadrilaterals



$p = 2, \Delta l = 0.01$ / Unstructured, triangles



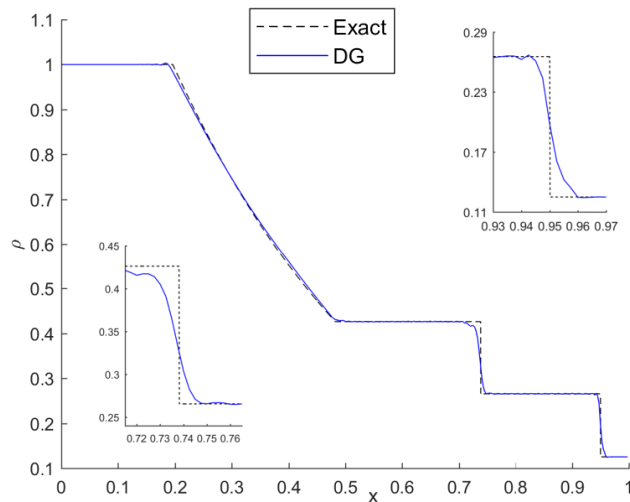
$p = 2, \Delta l = 0.01$ / Structured, triangles



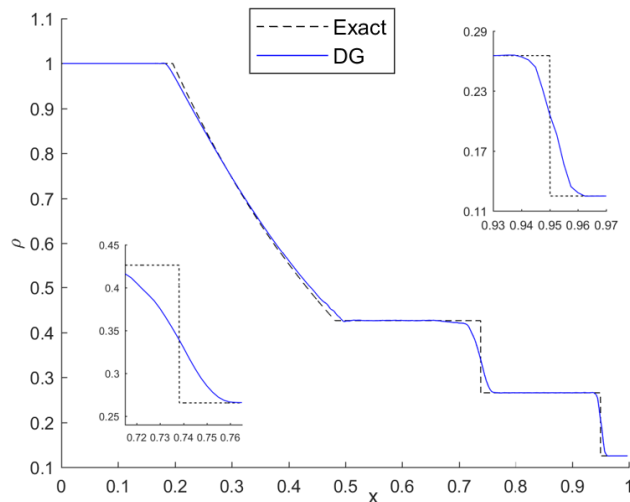
1D / $p = 2, nElem = 100$

Results

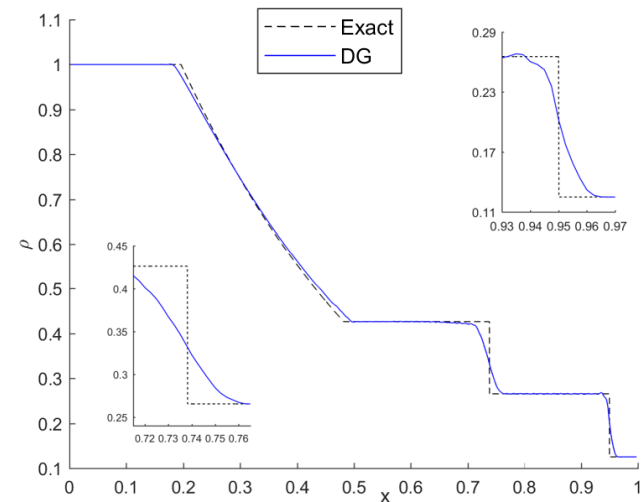
○ 2D shock tube problem



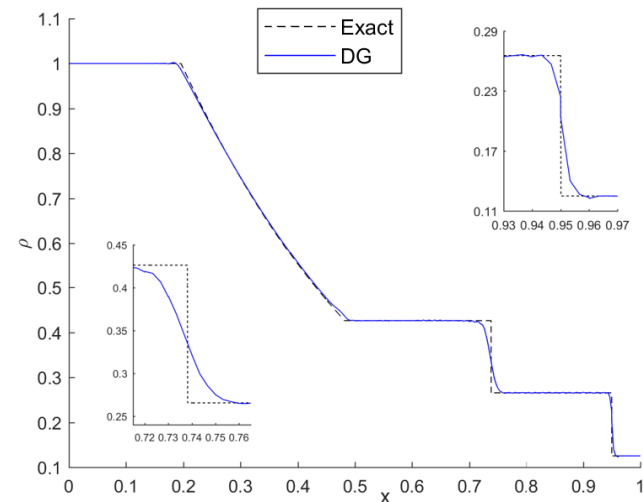
$p = 3, \Delta l = 0.01$ / Structured, quadrilaterals



$p = 3, \Delta l = 0.01$ / Unstructured, triangles



$p = 3, \Delta l = 0.01$ / Structured, triangles

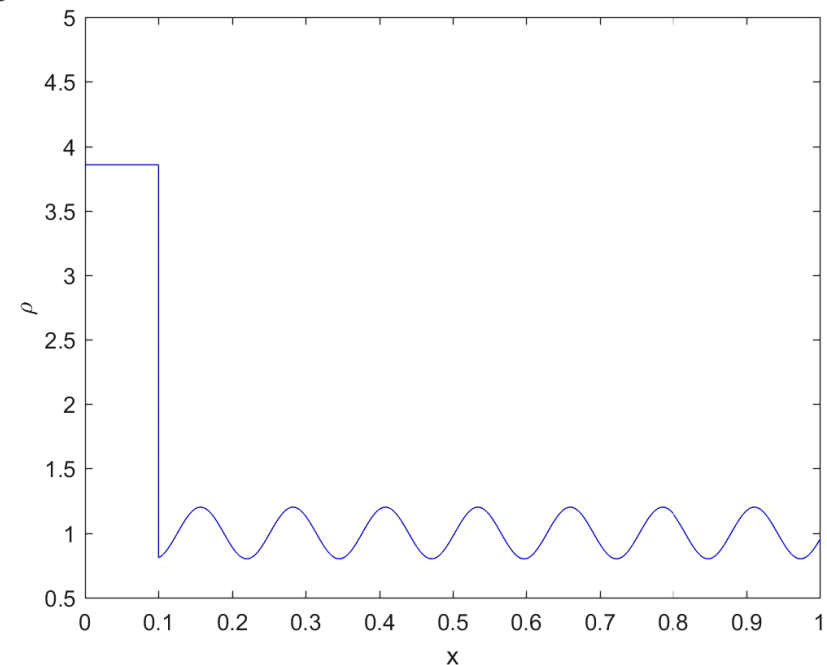


1D / $p = 3, nElem = 100$

Results

○ 2D Shu-Osher problem [1]

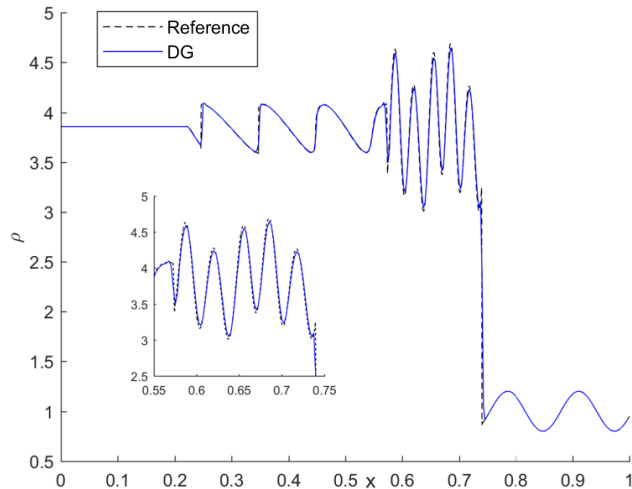
- Physical domain is divided into a high pressure part on the left side and a sinusoidal wave of density on the right side of a diaphragm
- A Mach 3.0 shock wave propagates to the right and interacts with the sinusoidal density profile
- $\Omega = [0, 1] \times [0, 0.1]$. The domain is discretized with triangles or quadrilaterals of characteristic length 0.0025
- CFL = 0.3
- Each simulation runs until $t = 0.18$



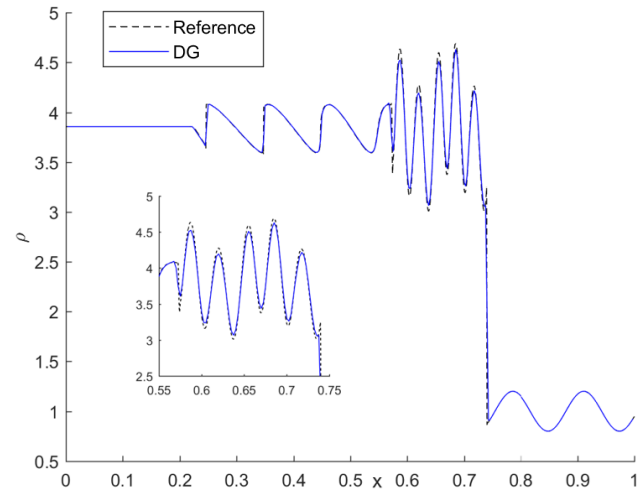
[1] Shu, C. W., and Osher, S., "Efficient implementation of essentially non-oscillatory shock-capturing schemes II," Journal of Computational Physics, Vol. 83, 1989, pp. 32–78.

Results

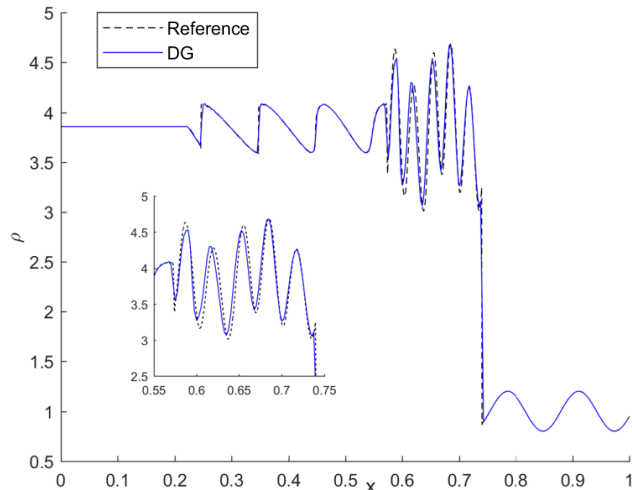
○ 2D Shu-Osher problem



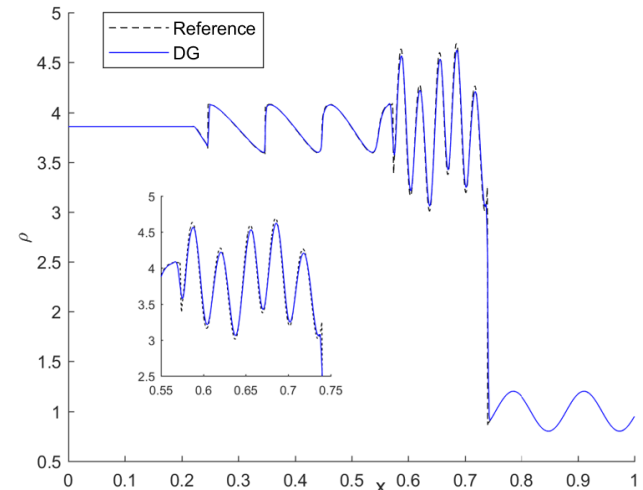
$p = 2$, $\Delta t = 0.0025$ / Structured, quadrilaterals



$p = 2$, $\Delta t = 0.0025$ / Structured, triangles



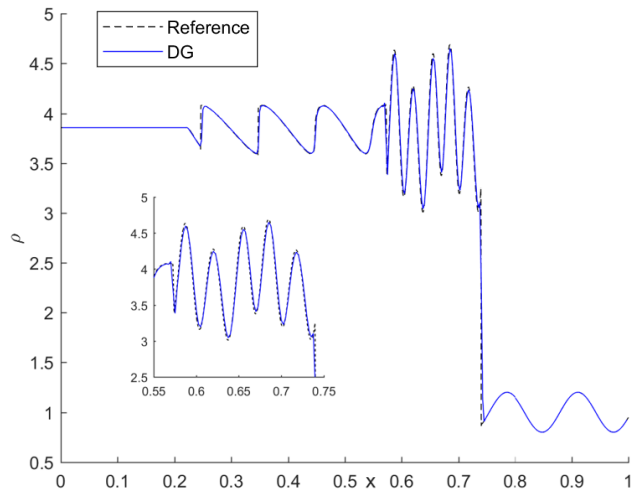
$p = 2$, $\Delta t = 0.0025$ / Unstructured, triangles



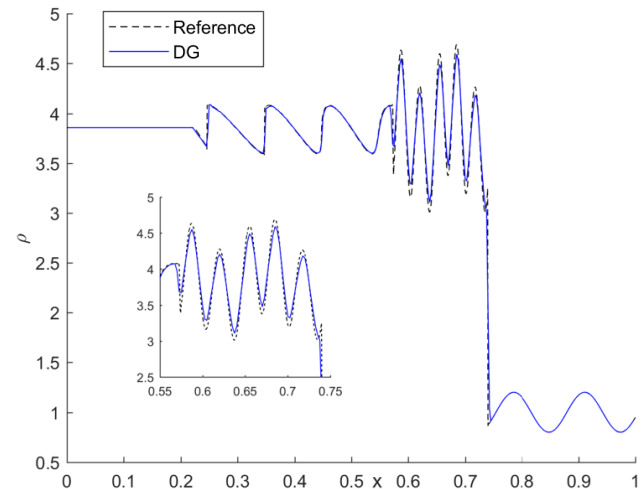
1D / $p = 2$, nElem = 400

Results

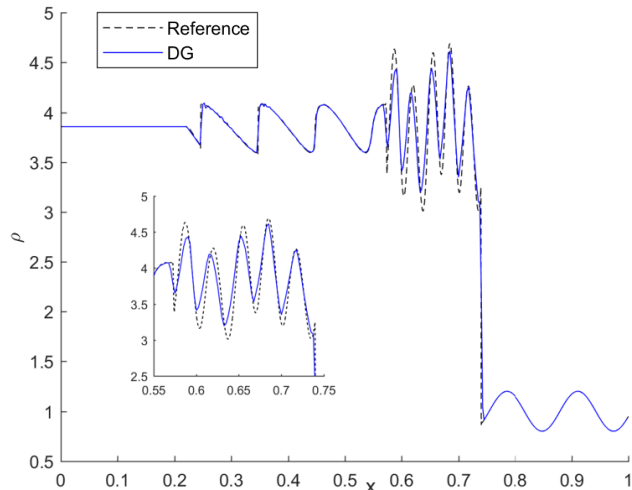
○ 2D Shu-Osher problem



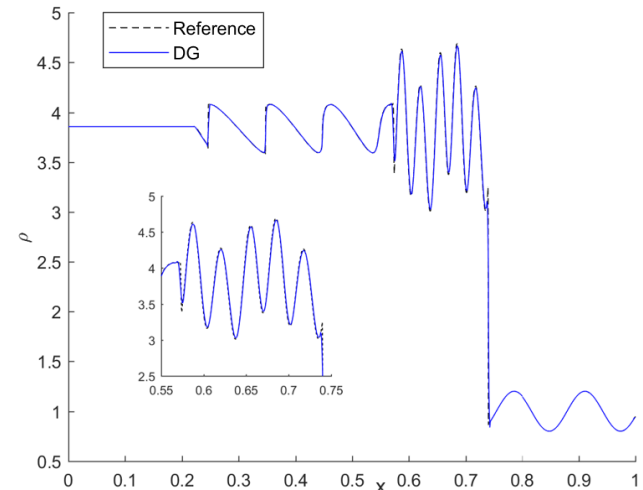
$p = 3, \Delta t = 0.0025$ / Structured, quadrilaterals



$p = 3, \Delta t = 0.0025$ / Structured, triangles



$p = 3, \Delta t = 0.0025$ / Unstructured, triangles

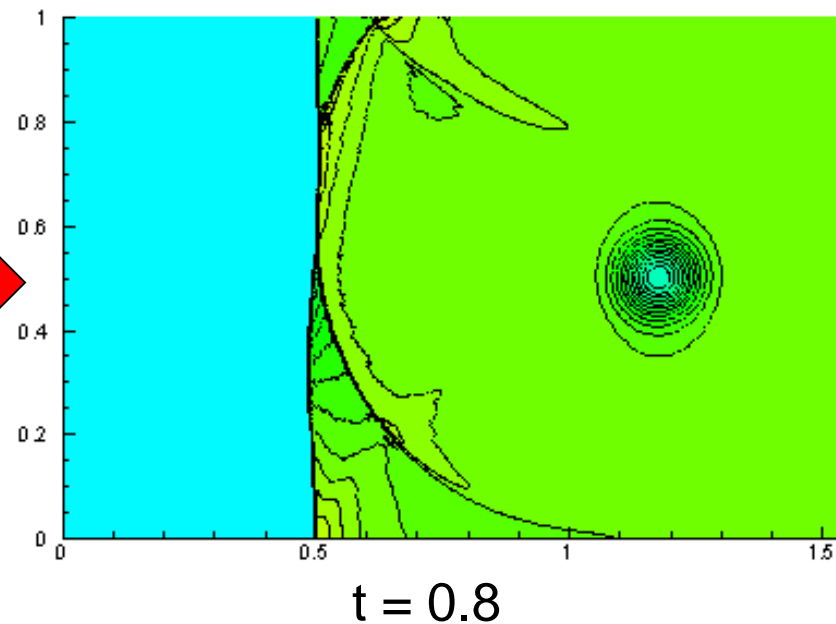
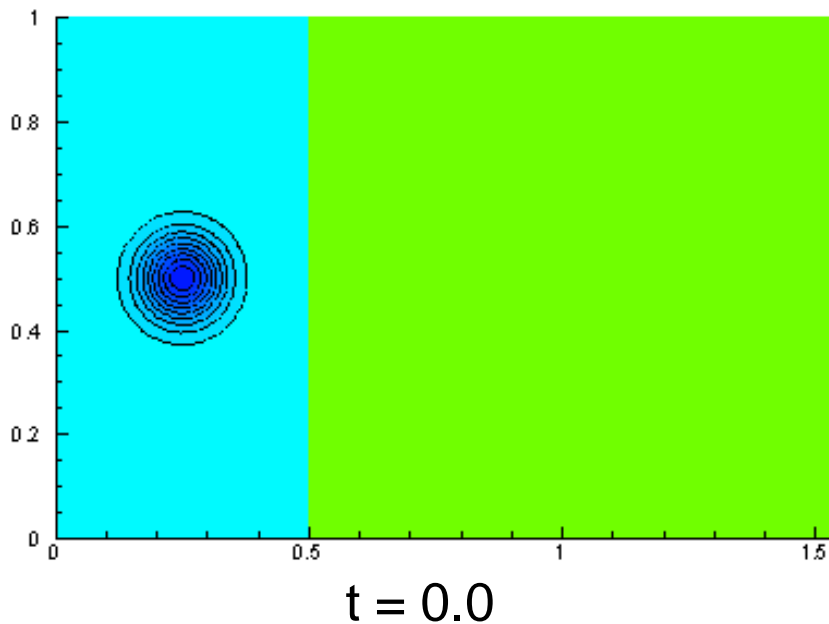


1D / $p = 3, nElem = 400$

Results

○ 2D Shock-Vortex Interaction^[1]

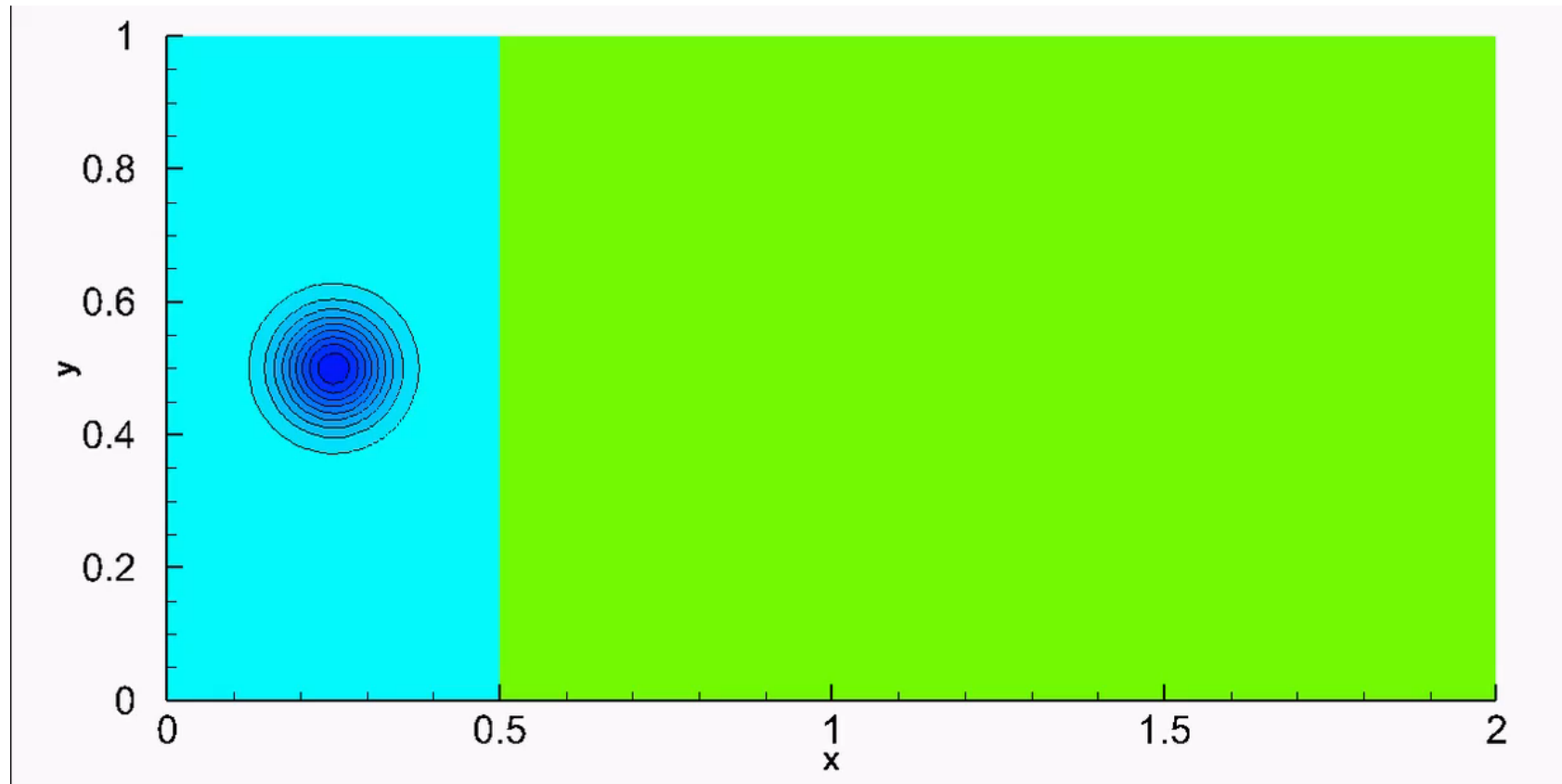
- A Mach 1.1 stationary shock interacts with a right running vortex.
- $\Omega = [0, 2] \times [0, 1]$. The domain is discretized with quadrilaterals.
- CFL = 0.1
- Each simulation runs until $t = 0.8$



[1] Jiang, G.S. and Shu, C.W., "Efficient Implementation of Weighted ENO Schemes," Journal of Computational Physics, 1996, pp.202–228

Results

- 2D Shock-Vortex Interaction

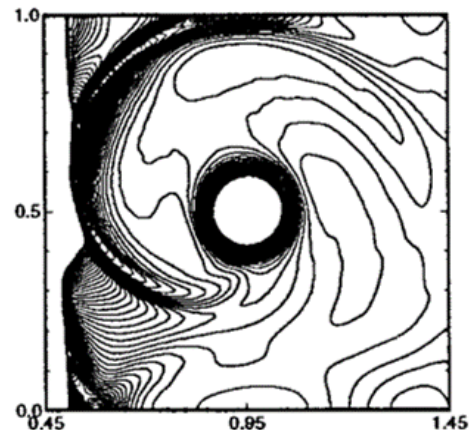


Choi, $p = 2$, LF, 200X100

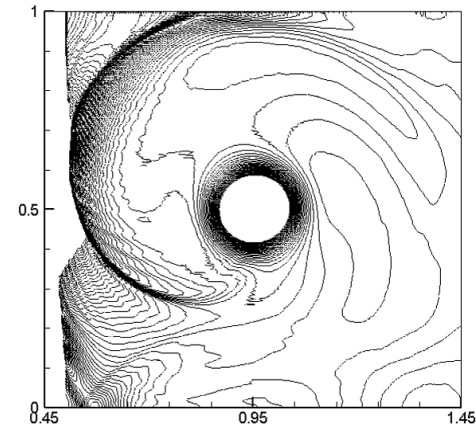
Results

○ 2D Shock-Vortex Interaction

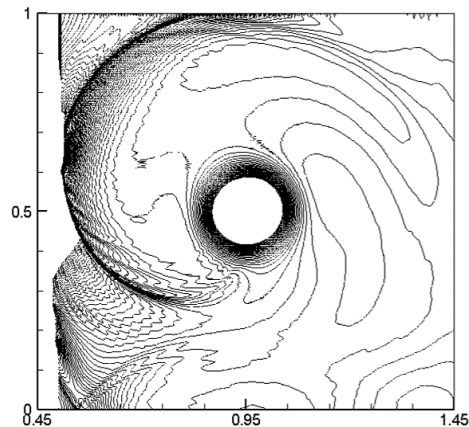
- $t = 0.6$, pressure, 90 contours from 1.19 to 1.37



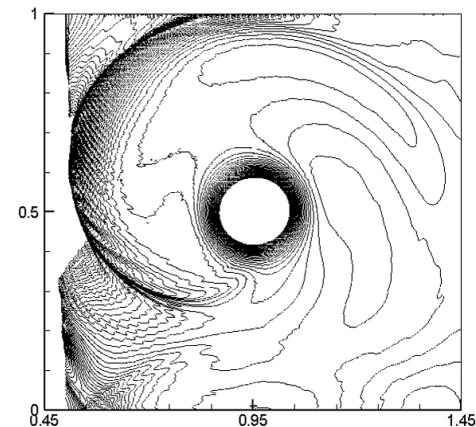
Jiang, WENO-LF-5, 251X100



Choi, $p = 2$, LF, 200X100



Choi, $p = 3$, LF, 150X100



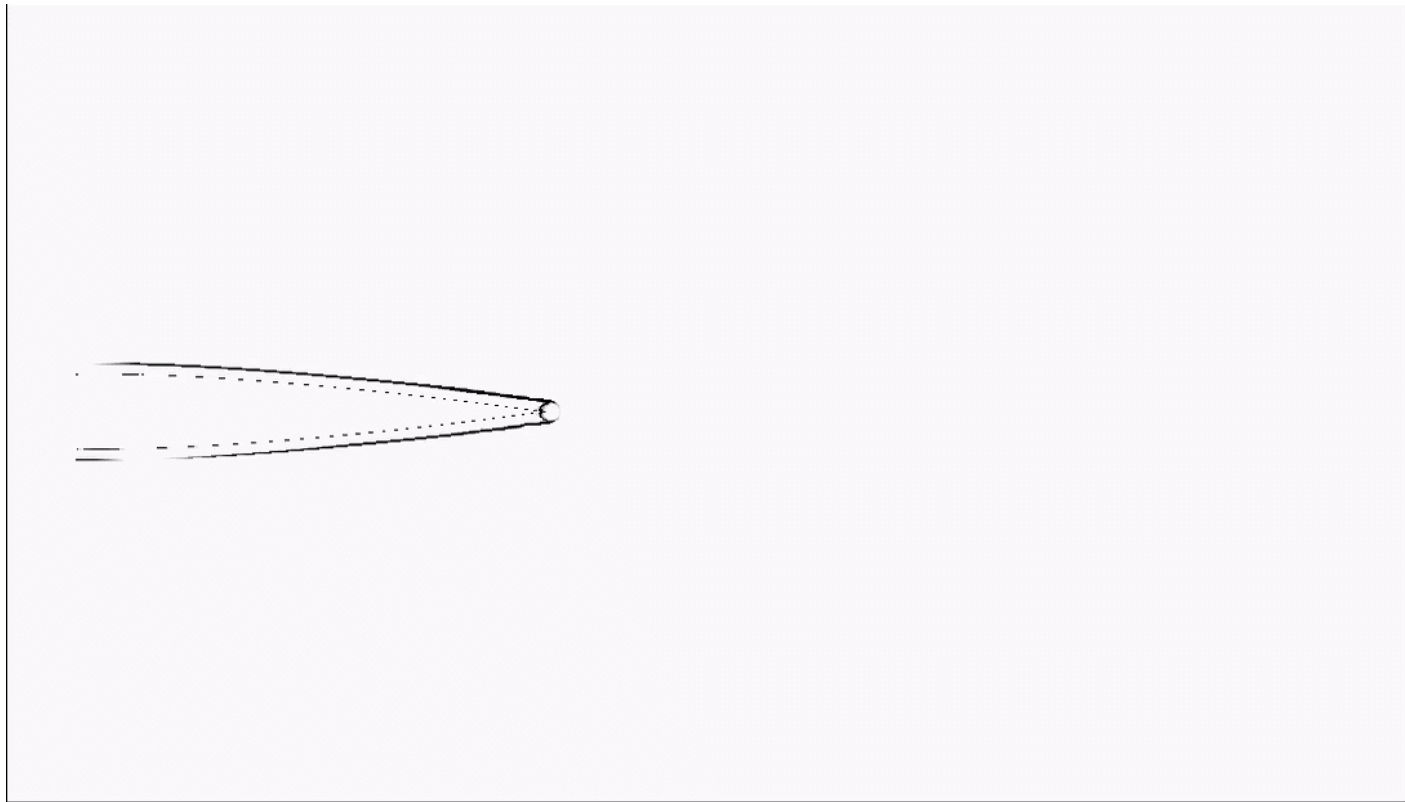
Choi, $p = 4$, LF, 120X100

Conclusions & Future Work

- A simple shock detector for the discontinuous Galerkin method is proposed in this work
 - The sensor only requires local-element information
 - The sensor is free from parameter tuning
 - The sensor operates well even for polynomial orders of 2 and 3 which are relatively low-order in typical DG implementations
- Functional relationship between the sensor and appropriate filtering strength is provided.
 - Shocks with various strengths can be captured
- Current investigations have extended procedure to 3D and are assessing the level of accuracy attainable
- Future efforts will be directed at suitability of procedure for accurate capturing of turbulence flows with shocks

Questions?

Thank you



Normalized Density Gradient Magnitude, SU2 DG-FEM solver

Navier-Stokes Eq., NACA0012, $Ma = 0.80$, $AoA = 1.25^\circ$, $Re = 60,000$, $p = 3$, 40K