Windowing Regularization Techniques for Unsteady Aerodynamic Shape Optimization

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10.06.2020

Unsteady periodic flows

Many fluid flows in industry are unsteady and turbulent.

- Aerodynamic design of rotor-craft
- Wind turbine design
- Flows around bluff bodies, e.g. airfoil at high angle of attack

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! URANS flow simulations for separated flows result in periodic flow fields.

! Flow field is smooth with clear periodicity.

(a) Snapshot at start of period (b) at 1=3 of period (c) and at 2=3 of period

Kármán vortex street behind airfoil) Undesirable!

Optimization objective in periodic flows

- Flow fields are often periodic after a transient time $t_{tr} = n_{tr}\Delta t$.
- Flow outputs such as Drag C_D or Lift C_L are periodic.
- Their sensitivities $\frac{\mathrm{d}}{\mathrm{d}}C_D$ and $\frac{\mathrm{d}}{\mathrm{d}}C_L$ w.r.t. design vector are periodic.

! Optimization objective: Outputs averaged over a period.

$$J(\ ; C_D) = \frac{1}{T(\ )} \int_{t_{tr}}^{T(\ ) \ t_{tr}} C_D(t; \ ) \, \mathrm{d}t \tag{1}$$

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- Period length $T(\ )$ unknown and dependent on !
- How to compute the sensitivity?

$$\frac{\mathrm{d}}{\mathrm{d}} J(\ ; C_D) = \frac{\mathrm{d}}{\mathrm{d}} \frac{1}{T(\ )} \int_{t_{tr}}^{T(\ ) \ t_{tr}} C_D(t; \ ) \, \mathrm{d}t \tag{2}$$

Finite time averaging

- Average over finite time M and hope for convergence as $M \to \infty$,

$$J_M(\ ; M; C_D) = \frac{1}{M} \int_{t_{tr}}^{M\ t_{tr}} C_D(t;\ )\ \mathrm{d}t: \tag{3}$$

- Similar for time averaged sensitivity

$$\frac{\mathrm{d}}{\mathrm{d}}J_M(\ ; M; C_D) = \frac{1}{M} \int_{t_{tr}}^{M\ t_{tr}} \frac{\mathrm{d}}{\mathrm{d}}C_D(t;\ )\ \mathrm{d}t: \tag{4}$$

Convergence properties

- Amplitude change in $\frac{\mathrm{d}}{\mathrm{d}} C_D$
- Increase M for convergence of $J_M(\ ; M; C_D)$ and $\frac{\mathrm{d}}{\mathrm{d}} J_M(\ )$?

[1] Wilkins, A., Tidor, B., White, J. and Barton, P. , "Sensitivity Analysis for Oscillating Dynamical Systems", *SIAM Journal on Scienti c Computing*, Vol. 31, No.4, 2009, pp. 2706-2732

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- Amplitude change in $\frac{\mathrm{d}}{\mathrm{d}} C_D$
- Increase M for convergence of $J_M(\ ; M; C_D)$ and $\frac{\mathrm{d}}{\mathrm{d}} J_M(\ )$?
- Fast growth in amplitude from sensitivity of phase and period length[1].
- Last values of $\frac{\mathrm{d}}{\mathrm{d}} C_D$ distort sensitivity results

$$J_M(\ )^{M\ !\ !^1} J(\ ), \text{ but } \frac{\mathrm{d}}{\mathrm{d}} J_M(\ )^{M\ !\ 6\ !\ !^1} \frac{\mathrm{d}}{\mathrm{d}} J(\ )$$

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Long time windowing by Krakos et al.[2]

- *!* Weighted average over finite time M and hope for convergence as $M \; ! \; 1$,

$$J_w(\;;M;C_D) = \frac{1}{M}\int_{t_{tr}}^{M+t_{tr}} w\left(\frac{t \quad t_{tr}}{M}\right) C_D(t;\;)\,\mathrm{d}t: \tag{5}$$

- Similar for time averaged sensitivity,

$$\frac{\mathrm{d}}{\mathrm{d}}J_w(\;;M;C_D) = \frac{1}{M}\int_{t_{tr}}^{M\quad t_{tr}} w\left(\frac{t \quad t_{tr}}{M}\right) \frac{\mathrm{d}}{\mathrm{d}}C_D(t;\;)\,\mathrm{d}t: \tag{6}$$

[2] Krakos, J., Wang, Q., Hall, S. and Darmofal L. "Sensitivity analysis of limit cycle oscillations", *Journal of Computational Physics*, Vol. 231 (2012), 3228-3245

Long time windowing by Krakos et al.[2]

- Weighted average over finite time M and hope for convergence as $M \to \infty$,

$$J_w(\ ;M;C_D) = \frac{1}{M}\int_{t_{tr}}^{M+t_{tr}} w\left(\frac{t-t_{tr}}{M}\right) C_D(t;\ )\,\mathrm{d}t: \tag{5}$$

- Similar for time averaged sensitivity,

$$\frac{\mathrm{d}}{\mathrm{d}}J_w(\ ;M;C_D) = \frac{1}{M}\int_{t_{tr}}^{M\ t_{tr}} w\left(\frac{t-t_{tr}}{M}\right) \frac{\mathrm{d}}{\mathrm{d}}C_D(t;\ )\,\mathrm{d}t: \tag{6}$$

- $w(\ ) \in C^k$, $= t/M$ is the windowing function with

$$\int_0^1 w(\ )\,\mathrm{d}\ = 1 \quad \text{and} \quad w(\ ) = 0; \quad \in \mathbb{R}\, n\,(0;1): \tag{7}$$

- $w = 1_{(0;1)} \in C^{k=\ 1}$ denotes a piecewise continuous window. (Finite time averaging is just a special case!)

[2] Krakos, J., Wang, Q., Hall, S. and Darmofal L. "Sensitivity analysis of limit cycle oscillations", *Journal of Computational Physics*, Vol. 231 (2012), 3228-3245

Convergence of long time windowing

- Convergence order depends on the smoothness of the windowing function $w \ 2 \ C^k$.

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Convergence of long time windowing

- Convergence order depends on the smoothness of the windowing function $w \in C^k$.

Theorem (Krakos et al. [2])

Let h be the period normalized flow output and $w \in C^k$. The windowed average error bound is given by

$$|J_w(\cdot;M) - J(\cdot)| \leq \|h\|_1 \, O(M^{-p}); \tag{8}$$

$$\left|\frac{\mathrm{d}}{\mathrm{d}}J_w(\cdot;M) - \frac{\mathrm{d}}{\mathrm{d}}J(\cdot)\right| \leq \|w\|_\infty \|h\|_1 \, O(M^{-p}) + \frac{1}{T}\frac{\mathrm{d}T}{\mathrm{d}}\|w\|_1 \|\dot{h}\|_1 \, O(M^{-(p-1)}) \tag{9}$$

where

$$p = \begin{cases} 1; & k = 1; \\ k+1; & k \neq 0; k \text{ even}; \\ k+2; & k > 0; k \text{ odd}. \end{cases} \tag{10}$$

! No convergence for traditional finite time average given!

[2] Krakos, J., Wang, Q., Hall, S. and Darmofal L. "Sensitivity analysis of limit cycle oscillations", *Journal of Computational Physics*, Vol. 231 (2012), 3228-3245

Mathematical formulation of the optimization problem

$$\min \quad J_w(u(\ );\ ;M;C_D) \tag{11a}$$

$$s.t. \quad u^n = G\ u^n;\ ;u^{n\ 1};u^{n\ 2}\ ; \qquad n = 1;:::;N \tag{11b}$$

$$J_w(u(\ );\ ;M;C_L)\quad c\ 2\ \mathbb{R} \tag{11c}$$

- Discretize windowed objective with midpoint rule,

$$J_w(C_D)\quad \frac{1}{N\quad n_{tr}} \sum_{n=n_{tr}}^{N} w\ \frac{n\quad n_{tr}}{N\quad n_{tr}}\ C_D(u^n;\ ): \tag{12}$$

- Flow constraint in fixed point formulation with 2nd order BDF method,

$$u^n = G(u^n;\ ;u^{n\ 1};u^{n\ 2}): \tag{13}$$

- Inequality constraints as windowed lift,

$$J_w(u(\ );\ ;M;C_L)\quad c: \tag{14}$$

Lagrange approach

- Compute descent direction $\frac{\mathrm{d}}{\mathrm{d}} J_w$ for optimization.
- Avoid expensive gradients like $\frac{\mathrm{d}}{\mathrm{d}} u^n$, that appear using the chain rule.

! Lagrange Function reads

$$L([u^1;\dots;u^n]^T;\ ;[\overline{u}^1;\dots;\overline{u}^n]^T) = J_w(u;\ ;M;C_D) + \sum_{n=1}^{N} (\overline{u}^n)^T\ \ G\ \ u^n;\ \ ;u^n\ \ ^1;u^n\ \ ^2\ \ \ \ u^n\ \ : \qquad (15)$$

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- Solve the KKT System

$$@_{\overline{u}^n} L = 0; \qquad n = 1;\dots;N \quad \text{state equations} \tag{16a}$$
$$@_{u^n} L = 0; \qquad n = 1;\dots;N \quad \text{adjoint equations} \tag{16b}$$
$$@\ \ L = 0; \qquad \text{design equation} \tag{16c}$$

Adjoint Sensitivity Computation

- Adjoint equations (adjoint iterator), given by $@_{u^n} L = 0$,

$$\overline{u}^n_{p+1} = H \quad \overline{u}^n_p; \quad ; \overline{u}^{n+1}; \overline{u}^{n+2} \quad ; \qquad n = 1; : : : ; N: \tag{17}$$

Adjoint Sensitivity Computation

- Adjoint equations (adjoint iterator), given by $\partial_{u^n} L = 0$,

$$\overline{u}^n_{p+1} = H\left(\overline{u}^n_p;\ ;\overline{u}^{n+1};\overline{u}^{n+2}\ ;\right), \qquad n = 1;\dots;N; \tag{17}$$

- In detail, we have

$$\begin{aligned} \partial_{u^n} L = {} & \overline{u}^n + \left(\partial_{u^n} G^n\right)^T \overline{u}^n + \underbrace{\left(\partial_{u^n} G^{n+1}\right)^T \overline{u}^{n+1} + \left(\partial_{u^n} G^{n+2}\right)^T \overline{u}^{n+2}}_{\text{from } 2^{\text{nd}} \text{ order BDF method in direct iterator}} \\ & + 1_{f n\ \ n_{tr} g} \frac{1}{N\ \ n_{tr}} w\left(\frac{n\ \ n_{tr}}{N\ \ n_{tr}}\right)\left(\partial_{u^n} C_D(u^n;\ \ )\right)^T : \\ & \underbrace{\qquad\qquad}_{\text{independent of } \overline{u}^n} \end{aligned} \tag{18}$$

Adjoint Sensitivity Computation

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$$\overline{u}^n_{p+1} = H\ \ \overline{u}^n_p;\ \ ;\overline{u}^{n+1};\overline{u}^{n+2}\ \ ; \qquad n = 1;:::;N: \tag{17}$$

- In detail, we have

$$\begin{aligned} @_{u^n} L = \quad & \overline{u}^n + (@_{u^n} G^n)^T\, \overline{u}^n + \underbrace{@_{u^n} G^{n+1}}_{\text{from } 2^{\text{nd}} \text{ order BDF method in direct iterator}}{}^T\, \overline{u}^{n+1} + \underbrace{@_{u^n} G^{n+2}}_{\text{from } 2^{\text{nd}} \text{ order BDF method in direct iterator}}{}^T\, \overline{u}^{n+2} \\ & + 1_{fn\ \ n_{tr} g} \frac{1}{N\quad n_{tr}} w \quad \frac{n\quad n_{tr}}{N\quad n_{tr}} \quad (@_{u^n} C_D(u^n;\ \ ))^T : \\ & \underbrace{\qquad\qquad}_{\text{independent of } \overline{u}^n} \end{aligned} \tag{18}$$

- Contractivity of H^n? ! Given by contractivity of G^n,

$$k@_{\overline{u}^n} H^n k = \ \ (@_{u^n} G^n)^T \ \ = k(@_{u^n} G^n)k: \tag{19}$$

! Adjoint iterator H^n inherits convergence of direct iterator G^n regardless of the chosen window.

Adjoint Sensitivity Computation

- Design equation is given by $@\ L$,

$$@\ L = \sum_{n=0}^{N} 1_{fn\ \ n_{tr}g} \frac{1}{N\quad n_{tr}} w\quad \frac{n\quad n_{tr}}{N\quad n_{tr}}\ @\ C_D(u^n;\ \ ) + (\overline{u}^n)^T\ @\ G^n \tag{20}$$

! Windowing function $w\quad \frac{n\quad n_{tr}}{N\quad n_{tr}}$ only appears as a *seeding* factor. ! super cheap!

! Only one floating point operation difference to traditional adjoint.

! Easy to implement.

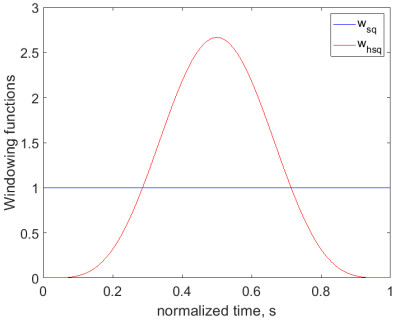
```
seeding = windowEvaluator.GetWndWeight(n,N,n_tr)/ (N-n_tr);
```

Test Case - NACA0012 Airfoil with high angle of attack[3]

- URANS Flow solver with JST flux.
- Spalart-Allmaras turbulence model with upwind flux.
- Reynolds number is 10^6, Mach number is 0:3.

Used windowing functions

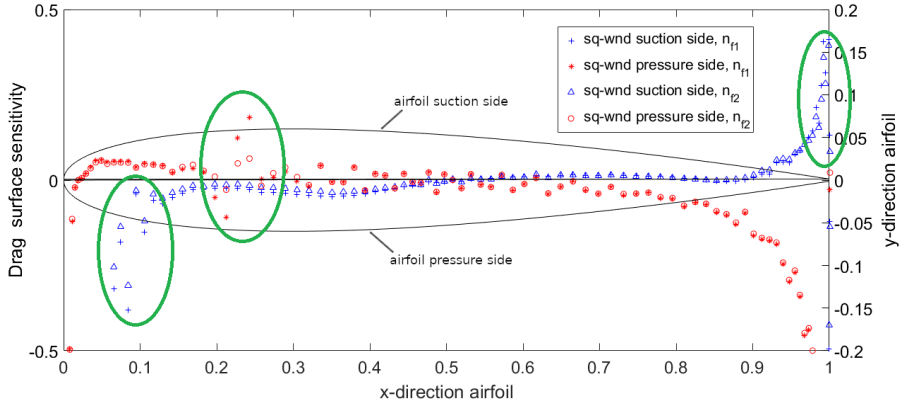
- Square: w_{sq} 2 C [1]
) $J_{w_{sq}}$ converges in $O(M$ [1]$)$
) @ $J_{w_{sq}}$ converges in $O(M^0)$
- Hann-Square: w_{hsq} 2 C^3
) $J_{w_{hsq}}$ converges in $O(M$ [5]$)$
) @ $J_{w_{hsq}}$ converges in $O(M$ [4]$)$
  Period boundaries are weighted less than the middle of the period.



[3] Resources online at https://su2code.github.io/tutorials/Unsteady_Shape_Opt_NACA0012/

Surface Sensitivities - Square Windowing

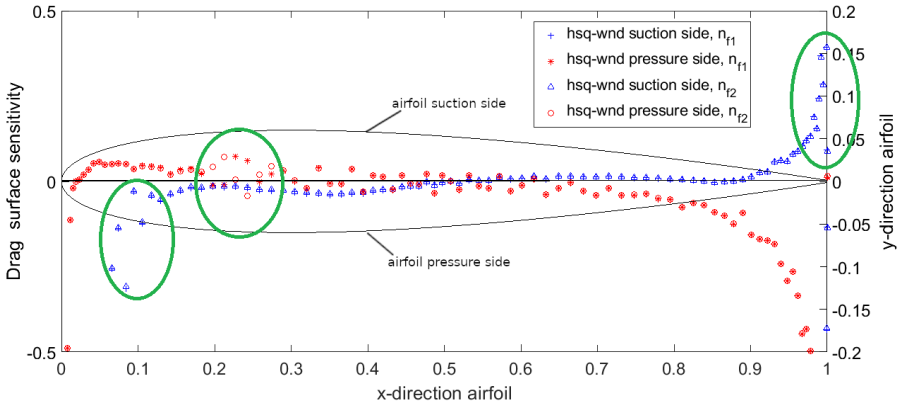
- Sensitivities of airfoil surface at final iterations n_{f1} and n_{f2}, where n_{f2} $n_{f1} = 1{=}3T$.



- Great difference of surface sensitivities for different final iterations!
- Square windowing is very sensitive w.r.t. choice of final iteration n_f.

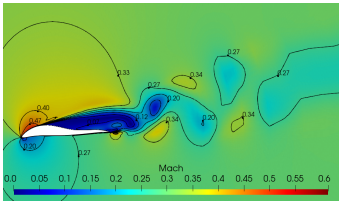
Surface Sensitivities - Hann-Square Windowing

- Sensitivities of airfoil surface at final iterations n_{f1} and n_{f2}, where n_{f2} $n_{f1} = 1{=}3T$.

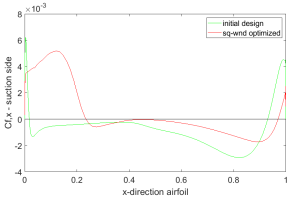


- Small difference of surface sensitivities for different final iterations!
- Hann-Square windowing is robust w.r.t. choice of final iteration n_f. ! Good for shape optimization!

Optimization Result - Square Windowing



(a) Initial design

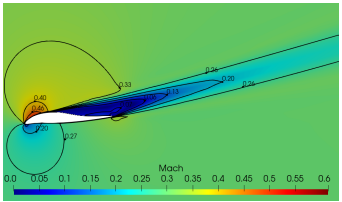


(b) After 15 design iterations

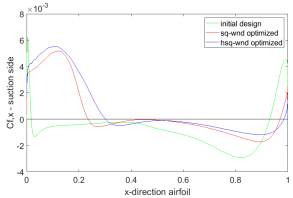
(c) Skin friction coefficient, x dir.

- Small reduction of the vortex street, still periodic behavior.
- $Cf\,;x$ has sign change at 23% airfoil length.
- The flow detaches itself further back on the wing than in the initial design.

Optimization Result - Hann-Square Windowing



(a) Initial design

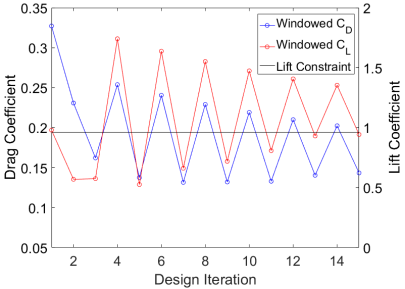


(b) After 15 design iterations

(c) Skin friction coefficient, x dir.

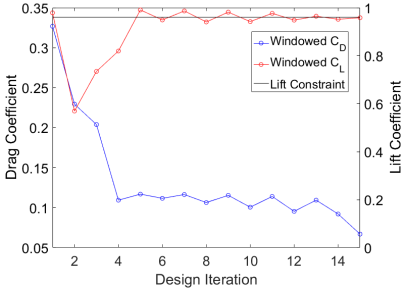
- Optimized flow is in steady state, no vortices left.
- $Cf\,;x$ has sign change at 30% airfoil length.
- The flow detaches itself 50% further back on the wing than in the Square windowed design.

Comparison of Design Processes



Square windowing

- ca. 59% drag reduction.
- Many designs infeasible (line search fails).
- Low Drag) low Lift.



Hann-Square windowing

- ca. 81% drag reduction.
- Almost all designs feasible.
- Low Drag, but high Lift.

) Similar results for other high order windows.

Windowing in SU2 - Usage and Implementation

- Two possibilities in SU2.
  - *!* Adjoint sensitivity computation via Lagrange approach and automatic differentiation.
  - *!* Windowed time averages of flow outputs or
    Windowed time averages of sensitivities via direct (tangent) sensitivity computation via AD (forward mode).
- Managed by Class `CWindowingTools` *!* Easy to add new windowing functions!

Windowing in SU2 - Adjoint mode and shape optimization[3]

- Define your windowing function `WINDOW_FUNCTION = HANN`
- Specify the final time iteration for the direct run `TIME_ITER = 1200`
- Specify the iteration to start the time average (after transient phase is over) `WINDOW_START_ITER = 500`
- Specify the iteration to start adjoint run `UNST_ADJOINT_ITER = 1200`
- Specify the window length M `ITER_AVERAGE_OBJ = 700`
- Choose the optimization objective `OPT_OBJECTIVE = DRAG * 1.0`

[3] Resources online at `https://su2code.github.io/tutorials/Unsteady_Shape_Opt_NACA0012/`

Windowing in SU2 - Direct mode[4]

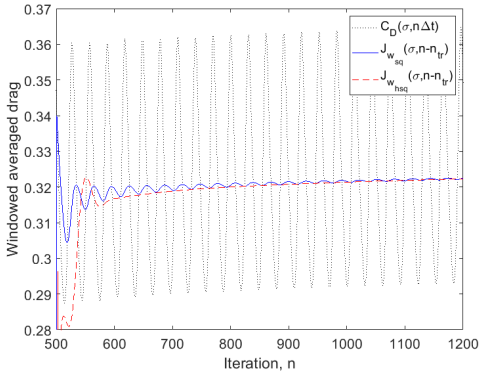
! Postprocessing step of the direct solver at each time iteration.
Compute $u^n = G(u^n; :::)$, then the *running* windowed average $J_w(u; n \quad n_{tr}; C_D)$.

- Define your windowing function `WINDOW_FUNCTION = HANN_SQUARE`
- Define the iteration to start the time average (after transient phase is over) `WINDOW_START_ITER = 500`
- Choose output groups/fields to average by using the prefix `TAVG_` (time average) and `D_TAVG_` (time averaged sensitivity)
  `SCREEN_OUTPUT = (TAVG_DRAG,D_TAVG_DRAG,...)`
  `HISTORY_OUTPUT = (TAVG_AEROCOEFF,D_TAVG_AEROCOEFF,...)`
- Value is the windowed average from `WINDOW_START_ITER` up to the current iteration.
  ! Can be used for time convergence monitoring!

[4] Resources online at `https://su2code.github.io/tutorials/Unsteady_NACA0012/`

Windowing in SU2 - Direct mode convergence monitoring[4]

- Idea: Output values should be a Cauchy sequence.
- Activate the window convergence criterion.
  `WINDOW_CAUCHY_CRIT = YES`
- Specify convergence field(s).
  `CONV_WINDOW_FIELD = (TAVG_DRAG, D_TAVG_LIFT)`
- Specify the number of elements to apply the criterion.
  `CONV_WINDOW_CAUCHY_ELEMS = 100`
- Set Epsilon to control the series convergence.
  `CONV_WINDOW_CAUCHY_EPS = 1E-2`



Convergence of windowed Averages

[4] Resources online at `https://su2code.github.io/tutorials/Unsteady_NACA0012/`

Windowing in SU2 - Efficient shape optimization workflows

Simulation

- Get end of transient phase n_{tr} (plus some buffer iterations).
- Save restart files, to avoid computing the transient phase in each design iteration.

Direct Differentiation

- Use windowed time convergence monitoring for sensitivity of objective function.
- Get minimal number N_{min} of time iterations needed for a converged windowed sensitivity.

Shape optimization

- Use restart files to skip transient phase in each design iteration.
- Use minimal time frame (plus some buffer iterations) for discrete adjoint computation, i.e. $M = (N_{min} + buffer) - (n_{tr} + buffer)$ and save computational time.

Summary and Conclusion

- Traditional time averages fail to compute sensitivities correctly in periodic flows.
- Long time windowing yields faster convergence of period averages and correct sensitivities.
- Windowing is easy to embed in adjoint sensitivity solvers.
- Windowing gives more robust surface sensitivities with respect to changing final times.
- Windowed shape optimization processes converge faster and are able to fulfill design constraints.
  ! The traditional approach fails to do this.
- Future work: Extension to DES flows. DES flows resolve multiple vortex scales.
  ! Decompose flow field into base frequencies and apply windowing.

Thank you for your attention!