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Implementation of a Modal Analysis Platform for Aeroelastic Computation in an Open Source CFD Solver SU2 and Application in Reduced Order Modelling

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The aim of this work is to implement the modal analysis platform for aeroelastic computation in the Open-Source CFD Solver SU2 for the application of linear Reduced Order Modelling (ROM). Although the current version of SU2 solver can handle periodic six degrees of freedom rigid movement of a solid surface immersed in a compressible flow, it cannot handle modal deflections or random excitations of a three-dimensional wing. This limitation is addressed in the present work in which the governing structural equations are assumed to be linear which implies that the mass and stiffness matrix associated with the linear structural equations is constant. To develop a linear reduced-order model, the structural modes need to be computed first. The CFD solver needs to be excited randomly using a Gaussian noise function in all the modal directions to facilitate the computation of the corresponding generalized aerodynamic forces. A subspace identification based reduced model is used for the construction of the linear reduced order models. The application of this approach on a clean wing of aspect ratio 10 of a rectangular planform composed of a NACA 0015 wing section is demonstrated for which SU2 source codes are modified to allow the wing to deform in the modal direction and generalized aerodynamic force vector is computed online for further application of linear reduced order model.

KEYWORDS
Reduced Order Model, SU2

1 | INTRODUCTION

Flexible aerodynamic components such as aircraft wings and control surfaces such as ailerons, flaps and slats undergo complex fluid-structure interaction resulting in aeroelastic instabilities such as flutter and Limit Cycle Oscillation (LCO). Reduced order modelling as outlined in Lucia et al. [1] has played a significant role in computational aeroservoelasticity since the last decade and has been effective for facilitating quick prediction of flutter, gust load, LCO, shape optimization, uncertainty propagation and various other issues which influence aerodynamic design. The Proper Orthogonal Decomposition (POD) method combined with the Galerkin projection approach is one of the most popular linear intrusive ROM used in the aeroelastic community as demonstrated in Hall et al. [2] in which the POD/Galerkin method is applied to unsteady transonic aerodynamics for flutter boundary computation by linearizing the governing equations and projecting these on the POD basis for reduced order computation. The Harmonic Balance Method of Thomas et al. [3] retains nonlinearity in the reduced system implying its effectiveness for computing the LCO. In this approach, the flow variables are expressed as Fourier series which facilitates an efficient computation of the periodic LCO. Among the non-intrusive ROMs, linear and nonlinear approaches are available for the flutter and LCO computation. The non-intrusive ROMs based on the Volterra theory by Silva et al. [4] and Balajewicz et al. [5], Neural Networks approach by Mannarino et al. [6] and Kriging interpolation by Timme et al. [7] are used for the computation of nonlinear aeroelastic phenomena like LCO. Likewise, there are several linear ROMs available in the non-intrusive platform. For the flutter boundary computation, the Eigen System Realization (ERA) method of Juang and Pappa [8] computes a linear state space system for the aerodynamics from the unit impulse response. The Subspace Identification based approaches of Overschee and Moor [9] compute the linear systems from the statistically independent signals. Halder et al. [10, 11], have shown nonintrusive linear and nonlinear reduced order model for two-dimensional airfoils for aeroelastic computations in SU2 [12]. For a three dimensional wing, the excitation of the wing in the pitch and plunge direction is not sufficient to compute the aerodynamic loads followed by reduced order model. Therefore, the wing needs to be excited in the generalized coordinate to compute the generalized aerodynamic forces. The contribution of the current work is the implementation procedure of the modal displacement of a wing immersed in compressible flow to compute the generalized aerodynamic forces associated with the aeroelastic computation in open-source CFD solver SU2.

2 | NUMERICAL METHODS

In this section, numerical methods for the fluid and structural solver will be discussed briefly to compute the high fidelity training data for the aerodynamic loads which will be further used for the development of the linear reduced order model.

2.1 | Flow Equations

The aerodynamic flows for this work are computed using the open source CFD solver SU2 which solves the unsteady compressible Navier-Stokes formulated on a mixed Eulerian or Arbitrary Lagrangian-Eulerian (ALE) reference frame. This is used for both the high-fidelity model and the training signals computation for the ROM. The ALE equations of the aerodynamic solver expressed in the weak formulation as

$$\iiint_{\Omega(t)} \frac{\partial W}{\partial t} dV + \oiint_{\partial\Omega(t)} (F^c_{ALE} - F^v)\hat{n} \cdot dS = 0 \;\text{ in }\; \Omega \times [0, t] \tag{1}$$

where W is the vector of flow variables, F^c_{ALE} and F^v are the convective and the diffusive terms respectively defined as

$$W = \begin{bmatrix} \rho \\ \rho\vec{v} \\ \rho E \end{bmatrix}, F^c_{ALE} = \begin{bmatrix} \rho(\vec{v} - \vec{v}_{airfoil}) \\ \rho\vec{v} \bigotimes (\vec{v} - \vec{v}_{airfoil}) + \bar{\bar{I}}p \\ \rho E(\vec{v} - \vec{v}_{airfoil}) + p\vec{v} \end{bmatrix}, F^v = \begin{bmatrix} 0 \\ \bar{\bar{\tau}} \\ \bar{\bar{\tau}} \cdot \vec{v} + \mu_{tot} C_p \nabla T \end{bmatrix} \tag{2}$$

where ρ is the fluid density, $\vec{v}$ is the velocity vector, p is the pressure field. T is the temperature field.and E is total energy per unit mass, $\bar{\bar{\tau}}$ is the viscous stress tensor, μ_{tot} consists of both the laminar and turbulent viscosity, C_p is the specific heat. A vertex-centered finite volume approach is used for the discretization of the governing equations. A spring analogy-based solver facilitates the mesh movement. The Geometric Conservation Law (GCL) is also enforced in the solver. In the present work, the viscous effect is ignored and the unsteady Euler equation is considered for the computation of the aerodynamic load.

2.2 | Structural Equations

The structural equations of motion of a three-dimensional wing model with n degrees of freedom and with aerodynamic loads can be expressed as follows

$$M\frac{d^2q}{dt^2} + C\frac{dq}{dt} + Kq = F_{aero} \tag{3}$$

where M, C, K are the structural mass, damping and stiffness matrices, q is a $n \times 1$ column vector representing the degrees of freedoms for each grid arising from the structural finite element model and F_{aero} is the aerodynamic force and moment vector which forms the applied loading. For this study, the structural damping is ignored i.e. $C = 0$. To execute the general modal analysis to compute the natural frequencies and the corresponding mode shapes of the configuration in the absence of the applied loading i.e. $F_{aero} = 0$, a reduced form of Eqn. (3) is generated by assuming a harmonic solution to permit amplitude changes while preserving the shape of the configuration. This assumption results in the following equation i.e.

$$(k - \lambda_i M)\phi_i = 0 \tag{4}$$

where $\lambda_i = \omega_i^2$ is the i^{th} real eigenvalue, ω_i is the circular frequency of the harmonic motion and ϕ_i is the i^{th} real eigenvector. Eqn. (4) is a set of homogeneous algebraic equations which are solved for the real eigenvalues for extracting the elastic modes by using the Lanczos method, and embodied in the SOL 103 algorithm in NASTRAN [13] which is used to compute the generalized mass and stiffness matrices and the mode shape matrix ϕ formed by combining the selected mode shape vectors i.e.

$$\phi = [\phi_1, \phi_2, ..., \phi_m] \tag{5}$$

where m is the number of the selected natural modes. Normally, the first four to six fundamental modes i.e. $m = 4$ to 6 are sufficient for constructing the generalized mass and stiffness matrices in the modal reference frame. The generalized mass matrix $\tilde{M}$, stiffness matrix $\tilde{K}$, and the generalized displacements vector η are computed as follows

$$\begin{aligned} \phi^T M \phi &= \tilde{M} \\ \phi^T K \phi &= \tilde{K} \\ \phi^T q &= \eta \end{aligned} \tag{6}$$

and the equation to generate the unforced structural reduced-order model can then be formulated as

$$\tilde{M}\eta + \tilde{K}\eta = 0 \tag{7}$$

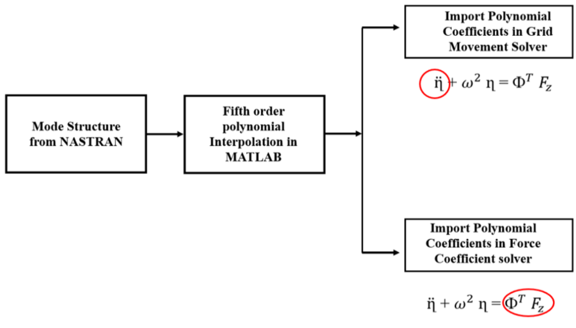
3 | IMPLEMENTATION IN SU2

In the current work, an open-source SU2 solver is used for the computation of the aerodynamic load. Since the linear structural equation is used for the aeroelastic computation, a third party structural solver NASTRAN is used, and the generalized mass and stiffness matrices are only conveyed to the SU2 based fluid solver. As implied in Eqn. (3), the structural equation and the fluid equation is projected on the generalized coordinates $\tilde{q}$. The mode shapes from Eqn. (7) are interpolated in a MATLAB [14] platform using the polynomial interpolations as follows

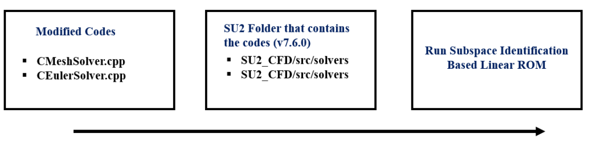
$$z = \sum_{i=k,...,0,j=0,...k} = Coef_{ij} x^i y^j \tag{8}$$

where z is the interpolated mode shape, $Coef_{ij}$ is the interpolation coefficient and x and y are the coordinates of the wing relative to a Cartesian coordinate system and k is the degree of the polynomial for the interpolation. The interpolation coefficient is added to the SU2 solver so that the aerodynamic surface moves in the modal directions instead of the rigid transformations following which the aerodynamic force and moments are computed in the force computation solver of SU2. For the computation of the generalized aerodynamic forces only the z directional force $F_{aero,z}$ is considered. The solver is modified by multiplying with the basis vectors ϕ and the generalized aerodynamic forces (GAF) are computed as follows

$$GAF_i = \phi_{i(x,y)} \times F_{aero,z}(x, y) = Coef_{ij} x^i y^j \times F_{aero,z}(x, y) \tag{9}$$



(a)



(b)

FIGURE 1 (a) Implementation in SU2 (b) Modification of the SU2 solver (Mesh movement solver and the Force computation solver)

The modification of the SU2 solver is shown in Fig. 1 and it shows that the generalized coordinate η is computed in the grid movement solver and the generalized aerodynamic forces are computed in the force computation solver inside SU2. The modification of the source codes and reduced order model codes are made available for public use in the GitHub link https://github.com/rahulhalderAERO.

A linear aerodynamic ROM as described by the following equation which is generated from the input random structural excitation and corresponding output GAF forces.

$$
\begin{aligned}
x_{ae}^{n+1} &= A x_{ae}^{n} + B u^{n} \\
y^{n+1} &= C x_{ae}^{n} + D u^{n}
\end{aligned}
\tag{10}
$$

where the constant matrices A, B, C and D are used to approximate the aerodynamic system . The detail s of the algorithmic steps of the constant system matrices formulation are described in Overschee and Moor [10]. u and y are input and output vectors and x_{ae} is the aerodynamic state vector.

4 | RESULTS AND DISCUSSIONS

This section discusses the implementation of the modal analysis platform for the aeroelastic analysis in SU2 for the application of the linear ROM of aerodynamic loads. The structural wing modes are first computed in the NASTRAN platform and the wing surface is excited randomly in the modal direction to compute the training aerodynamic loads

TABLE 1 Geometrical and Material properties of wing.

Geometrical Prop.	values	Material Prop.	values
Wingspan	4.5 m	Mass of Wing	2.1392 kg
Wing Root Chord	0.65 m	Young's Modulus E1	3.1511 GPa
Wing Tip Chord	0.25 m	Young's Modulus E2	0.4162 GPa
Wing Section Semi-Chord	0.45 m	Shear Modulus G	0.4392 GPa
Sweep-back Angle	0	Poisson Ratio	0.31
Wing Section	NACA0015	Density	381.98 kg/m^3

subsequently. The subspace identification based linear reduced order model is then further applied on the training data set for quick prediction of generalized aerodynamic load under any structural input excitation.

4.1 | Generation of the Structural Mode

The structural equations are solved in NASTRAN platform which is one of widely used structural analysis packages used by many researchers for various applications. The modal analysis platform is demonstrated in this section by considering an application to a high aspect ratio rectangular planform wing consisting of a NACA 0015 wing section, the geometrical and material properties of which are shown in Table 1. A tapered wing of aspect ratio 10 with a root chord of 0.65 and a tip chord of 0.25 is considered for the present computation.

The structural modes ϕ as shown in Eqn. (5) are interpolated using a fifth order polynomial function in MATLAB as shown in Figure 2. The R^2 fit of the interpolation is considered 0.99 for all the structural modes as defined by the following Eqn.

$$R^2 \text{ fit} = 1 - \frac{\sqrt{\sum (y_{predicted} - y_{truth})}}{\sqrt{\sum (y_{truth})}} \tag{11}$$

Figure 2(a) , 2(b) and 2(c) show the first bending , second bending and third bending mode respectively and Figure 2(d) shows the torsional mode. In Fig. 2(a) to (d) , the black dots of each image show the actual wing mode to be interpolated and the coloured part is the surface obtained from polynomial interpolation using the coordinates of the black dots.

The coefficients of the polynomial function shown in Eqn.(8) are then conveyed to the flow solver in SU2 to excite the wing in the modal directions which will be discussed below.

4.2 | Excitation of the Aerodynamic Surface in the Modal Directions

The wing surface is excited in the modal directions as shown in Figure 2 (a)-(d) and the corresponding generalized aerodynamic forces are computed using Eqn. (9) and as indicated in Fig. 1. First, the aerodynamic surface is excited randomly in the modal directions to generate the training dataset for the reduced order model and then excited with the sinusoidal input for the validation of the effectiveness of the reduced order model. For both random and sinusoidal excitation, the predicted GAFs are compared with the CFD based result. The wing deflections as a result of

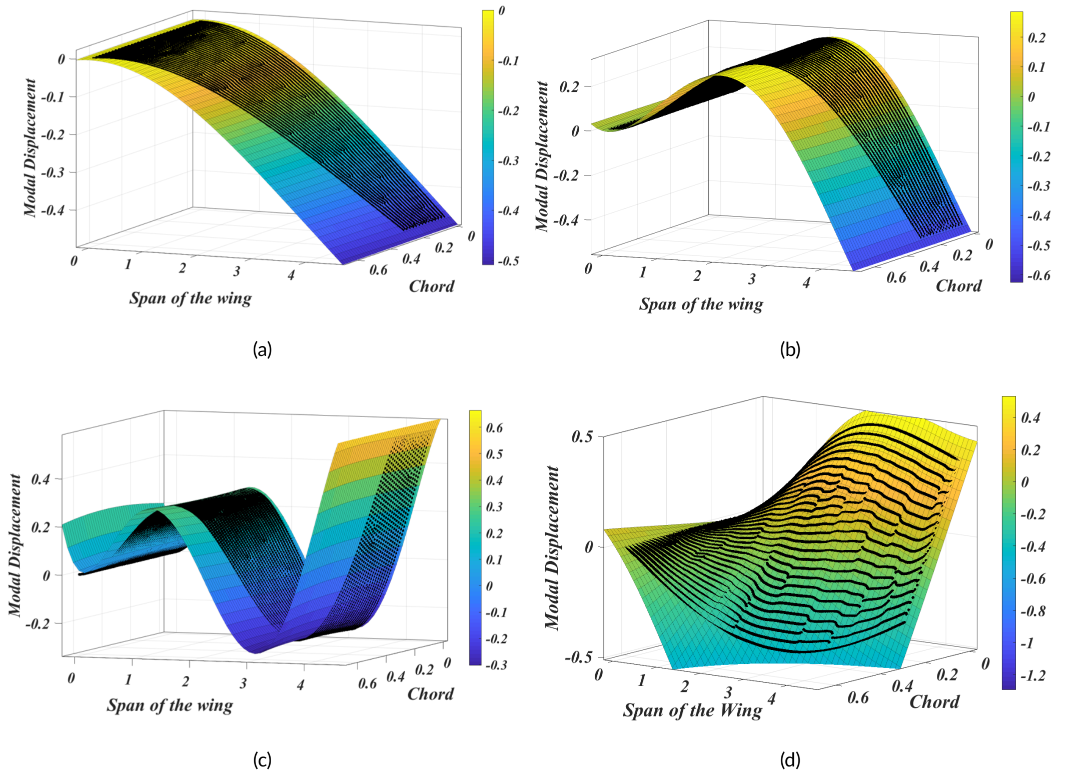
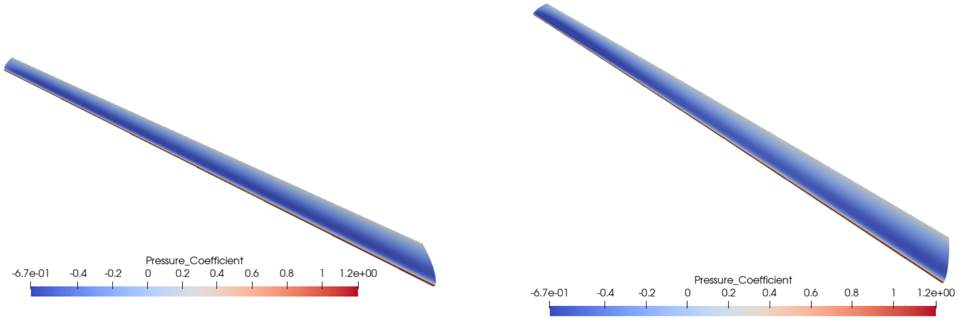


FIGURE 2 Interpolation of the structural modes in the MATLAB platform (a) First Mode (b) Second Mode (c) Third Mode and (d) Fourth Mode.

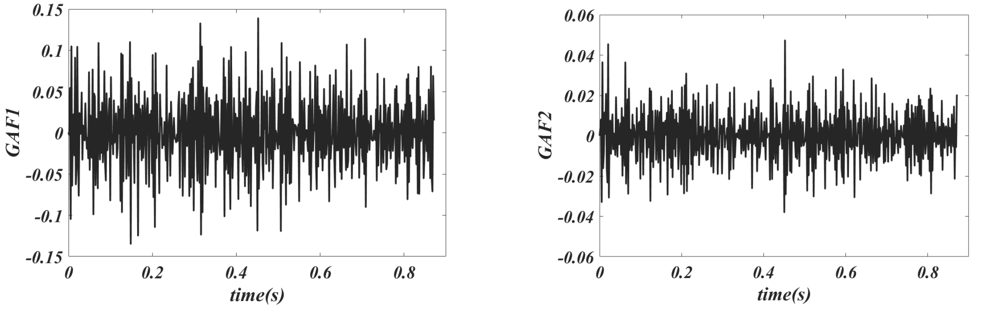
the sinusoidal excitation of the aerodynamic surface in the modal directions (modes I and II) with a structural input of $0.1sin(50t)$ are shown in Fig. 3. Figure 3(a) and 3(b) show the first and second mode respectively corresponding to the state excited at the peak of the sinusoidal excitation in each cycle. The computed wing surface Cp contours are shown at the excited location. To develop a linear reduced order model, the wing is now excited in the mode I direction under a random noise and generalized aerodynamic forces GAF1 and GAF2 are computed. Similarly, the wing is excited in the mode II direction under a different set of Gaussian noise and the generalized aerodynamic forces GAF1 and GAF2 are computed under those excitations. The generalized forces GAF1 and GAF2 under the excitation in mode I and mode II directions are finally added to compute the total GAF1 and GAF2 which will be used for the development of the linear ROM. Figure 4 shows the temporal variation of GAF1 and GAF2 under the combined excitation of the wing in the mode I and mode II direction. The number of states considered in the subspace identification algorithm is 15.

Since the number of inputs and outputs is 2, the size of the A matrix as shown in Eqn. (10) is $R^{30\times30}$. Since Eqn. (10) describes a discrete system and therefore the eigen distribution of constant matrix A should lie in the unit circle for the linear system to be stable. Figure 5 shows the stable eigenvalue distribution of the system matrix A.Figures 6(a)-(b) compares the linear ROM prediction of the GAF1 and GAF2 under random excitation in mode I while Figs. 6(c)-(d) compares the same for excitation in the mode II directions with predictions from CFD models. For better



(a) (b)

FIGURE 3 The wing deflected at the maximum sinusoidal location under (a) first and (b) second mode excitation.



(a) (b)

FIGURE 4 Generalized aerodynamic force (a) GAF1 and (b) GAF2 under combined mode I and mode II interaction.

visualization of the ROM application, the comparison is shown only for the window of the time t between 0.2 and 0.3 s. The linear aerodynamic ROM is then used to compute the aerodynamic forces under sinusoidal excitation of 0.01sin (50 t). Figures 7(a)-(b) shows the comparison of GAF1 and GAF2 under the sinusoidal excitation of mode I. Figure 7(c) and 7(d) shows the comparison of GAF1 and GAF2 under the sinusoidal excitation of mode II.

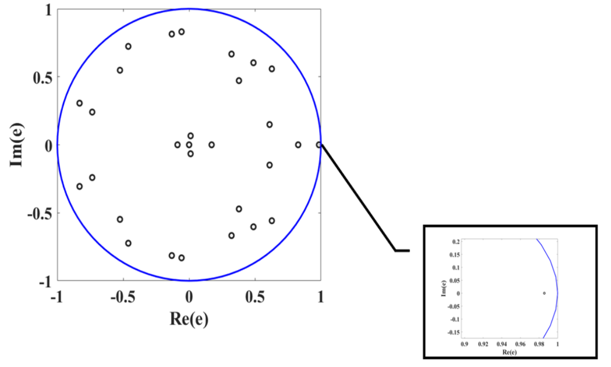


FIGURE 5 Eigenvalue distribution of the stable *A* matrix arising from the linear system developed for the wing aerodynamics.

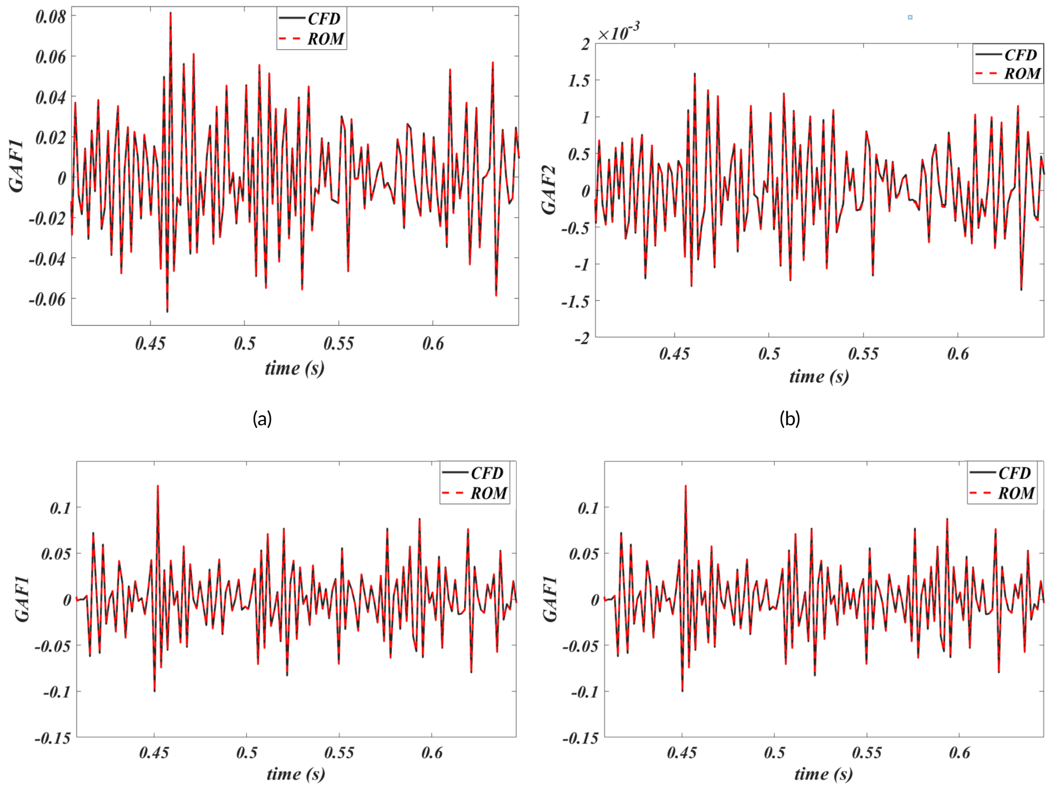


FIGURE 6 (a) GAF1 and (b) GAF2 under mode I excitation (c) GAF1 and (d) GAF2 under mode II excitation randomly.

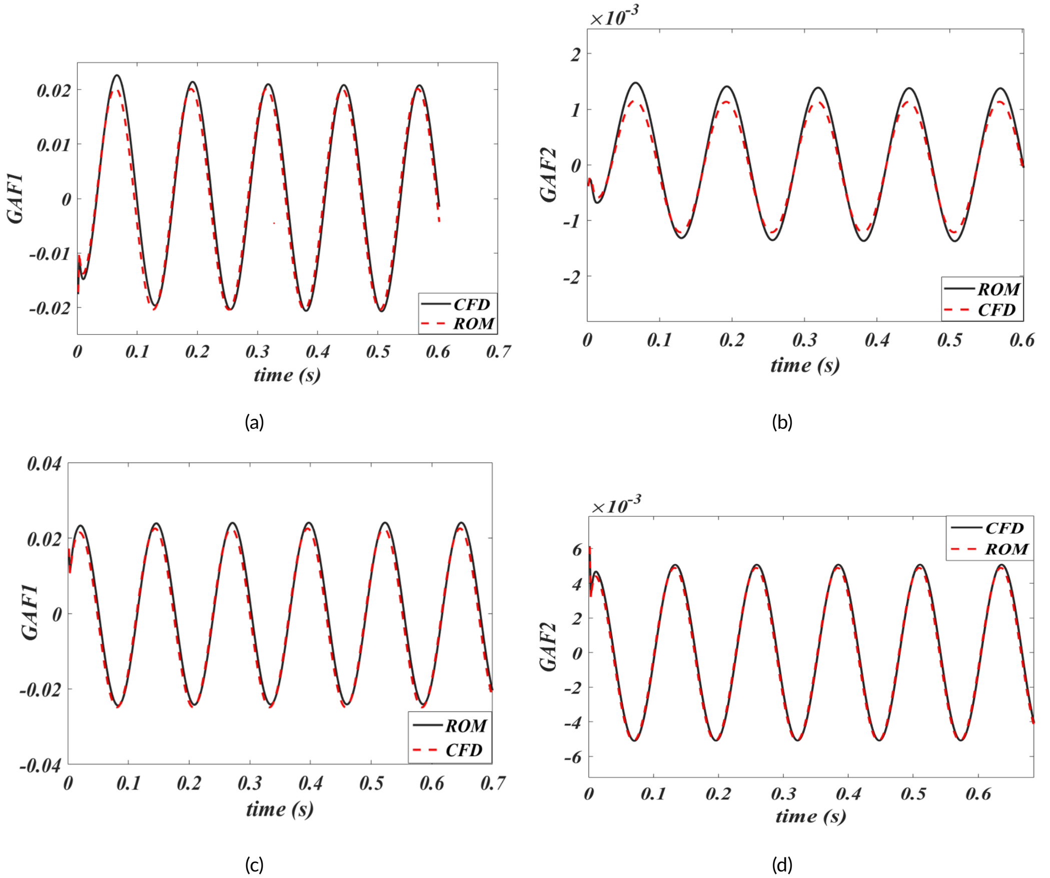


FIGURE 7 (a) GAF1 and (b) GAF2 under mode I excitation (c) GAF1 and (d) GAF2 under mode II excitation in sinusoidal direction.

5 | CONCLUSIONS

This work describes the implementation details of the modal excitation of an aircraft wing undergoing arbitrary motion in the modal coordinates instead of a rigid transformation in the SU2 solver and therefore, this development is very essential for aeroelastic computations. The work shows that polynomial functions of order five are sufficient for the interpolation of the structural modes and a fit value of 0.99 can be attained. The subspace identification based linear ROM can be applied to reconstruct the generalised aerodynamic forces. This effort shows that existing SU2 codes can be modified to handle the modal deformations of a wing which has a significant importance in aeroelastic applications.

Conflict of Interest

There is no conflict of interest.

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